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THE INFLUENCE OF ATMOSPHERIC HUMIDITY AND OTHER FACTORS UPON THE STATIC LIFT OF AIRSHIPS

BY

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Introduction

The following Paper comprises some of the results of an investigation originally undertaken by the writer to determine the effects of the humidity of the atmosphere and other secondary factors upon the static lift of airships. It is almost invariably found on completion of such a craft that a considerable deficiency is revealed between the designed lift and the apparent value deduced, for example, from a "lift and trim" trial. Clearly, it is of considerable importance that the reason for this discrepancy should, if possible, be determined; for whilst the loss may be only a small percentage of the total displacement of the ship, and hence difficult to trace, yet considered in terms of disposable lift, it frequently is very serious.

The deficiency may be due to a variety of causes, such as inaccurate determination of "dead space" and integrated fixed weights, but there has been for some time a strong suspicion that in fabric-covered ships a contributory one is the hygroscopic nature of the outer-cover and gas-bags, the absorption of moisture causing an increase of the weight of the latter cover over and above that allowed for in design. Accordingly, to investigate this point experimentally, various representative samples of fabric were prepared and exposed for long periods to conditions of temperature and humidity which simulated as far as possible those to which the fabric would be subjected in practice.

In addition, it is just possible that there may be small but accumulative errors in existing lift formulæ, due to the employment of assumptions in their establishment only approximately true and to the lack of exactness in certain fundamental definitions.

The writer has made it the main object of this Paper, therefore, to examine carefully the theory of such formulæ, to check numerically the error involved in any hypothesis, and eventually to deduce accurate equations (applicable by a simple change of constants to either hydrogen or helium as lifting agent) in a form suitable for computation by logarithms or slide-rule. This latter point is, in the writer's opinion, of especial importance in practical work, as existing formulæ, if they be exact and allow in particular for humidity effects, are not adapted to rapid calculation and are moreover usually inconsistent and confusing in the units employed.

The influence of moisture in the atmosphere and lifting agents is apparent not only in the change of density of the gases which it involves, and in the " soakage " of the fabric, but also, in a more general sense, by its dynamic effect. By this is meant the increase of apparent weight caused by the impact of rain-drops on the hull of an airship, and by the " free " water which collects there during precipitation. An investigation into all these phenomena is clearly desirable since there is an impression in certain quarters that their accumulated effect largely contributed to the disaster to the R.101.

In the future, more than ever in the past, airships will be called upon to proceed possibly from sheds in low-lying and foggy country, across arid deserts, over hot and humid watersheds and forests, and through tropical rainstorms. An analysis of the changes of lift which would be experienced due to these widely-varying conditions is obviously desirable in the interests of future design and of safety in travel.

The various important formulæ and other data useful in the design of airships are collected together in Appendix I, and in view of the great importance of accurate measurement of gas purity in such work there is a short discussion in Appendix II upon the various types of meter used commercially, together with some suggestions as to a new form of instrument, the principle of which is believed to be novel.

1. *The Gas Laws*

The behaviour of gases under atmospheric conditions of temperature and pressure, and when in a state far removed from their liquefaction point, can be shown experimentally to obey nearly exactly the laws of Boyle, Charles, Avogadro and Dalton. These are in order:

(*A*) The volume of a given mass of gas at constant temperature varies inversely as the pressure.

(*B*) The volume of a given mass of gas at constant pressure varies directly as the absolute temperature (*i.e.*, the temperature reckoned from " absolute zero," or -273°C., so that °A. $= 273 +$ °C.)

(*C*) All gases at the same temperature and pressure contain the same number of molecules per unit volume.

(*D*) In a mixture of gases which exert no physical or chemical action on one another, each one develops the same pressure as if it alone occupied the entire space and the total pressure is the sum of the partial pressures of each.

From these laws the so-called " Perfect-Gas " Equation is easily deduced, which is

$$pv = RT \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (i)$$

where p is the pressure of the gas
v ,, ,, volume of 1 mol of the gas
T ,, ,, absolute temperature ,, ,,
R ,, ,, universal gas constant, the same for all gases, and equal to 83.15×10^6 ergs per mol per °*A*. [1] (See Appendix I.)

One mol of a substance is M grams, where M is its molecular weight in the chemical sense. Thus the molecular weight of hydrogen (H_2) being 2×1.008 or 2.016, 1 mol of hydrogen is 2.016 grams; that of oxygen (O_2) being 2×16 or 32, 1 mol of oxygen is 32 grams. If the substances be, like dry air, a mixture of gases all of which obey the above laws, then M is the " average " molecular weight, and is found (see paragraph 2) by multiplying the parts by volume of each constituent by its corresponding molecular weight, summing up the products, and dividing by the total volume. Thus a mixture of 3 parts by volume of

[1] Kaye and Laby's " Physical and Chemical Constants," p. 5.

hydrogen and one part of oxygen would have an " average " molecular weight equal to $\{ (3 \times 2.016) + (1 \times 32) \} / 4$, or 9.512

Again, if a mass of 1 gram only of a gas be considered, the " Perfect Gas " Equation obviously becomes

$$pv = RT/M \quad . \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

The above relations are obeyed almost exactly by hydrogen, helium, and dry air under atmospheric conditions and will be used without further modification in the subsequent analysis. Some investigation is desirable, however, to determine how far they are applicable to water-vapour, which commonly exists in the atmosphere and in gas-containers, since its liquefaction point is normally very much nearer atmospheric temperatures.

If a given mass of fluid be kept at a constant temperature and its volume and pressure be altered, then the relation between the two latter quantities for various values of temperature is given graphically by " isothermals " such as in Fig. 1. The typical " isothermal " *a b* corresponding to a temperature well above liquefaction point obeys nearly exactly the relations (i) or (ii). If, however, we consider one such as *c d e f*, for a very much lower temperature, then whilst between *c* and *d* the substance is gaseous, at *d* it commences to liquefy. Between *d* and *e* the pressure remains constant as the volume is decreased, and the substance continually condenses until at *e* it is wholly liquid. For such cases it is usual to speak of the state of the substance between *c* and *d* as that of " unsaturated vapour," and the gaseous part of the mixture between *d* and *e* as " saturated vapour." The " saturated vapour pressure " corresponding to this last depends only on the temperature, increasing with the latter, and is independent of the quantity of fluid present and of the presence of any other gases (provided always that there is no physical or chemical action between them). At *e* the substance is completely liquid and any further compression only decreases its volume to a very small extent shown by *e f*.

It is found when the " isothermals " are plotted for various increasing temperatures, that the length of *d e* progressively decreases until for a particular " isothermal " corresponding to what is known as the " critical temperature " it vanishes; this is shown by the point *g* in Fig. 1. Above this temperature it is impossible to liquefy the substance, whatever pressure be applied, and the point *g* obviously corresponds to a " critical " volume and pressure.

The following values are given in Kaye and Laby's " Physical and Chemical Constants " p. 36.

	Critical Data.		
Substance.	Temperature in °C.	Pressure in Atmospheres.	Volume.
Hydrogen	−234.5	20	.00264
Helium	−268	2.3	.00299
Air	−140	39	.00468
Water	365	194.6	.00386

The " critical volume " is here defined as the ratio of the volume that a given mass of gas has at the " critical " temperature and pressure to that which it would have at 0°C. and under the normal atmospheric pressure of 1013.2 mb.

Since an " isothermal " such as *c d e f* is discontinuous at *d* and *e*, it is clearly impracticable to evolve an " equation of state " which shall represent for any given temperature the condition of the fluid for all values of pressure and volume. Such phenomena as " super-saturation " and " superheating " show, however, that if dust and ionic nuclei be excluded, also if the substance be kept free from mechanical disturbance or vibration (and in the case of a liquid from dissolved gases) then there is a tendency for such a path as *c d q r s e f* to be followed instead of *c d e f* (see Fig. 2).

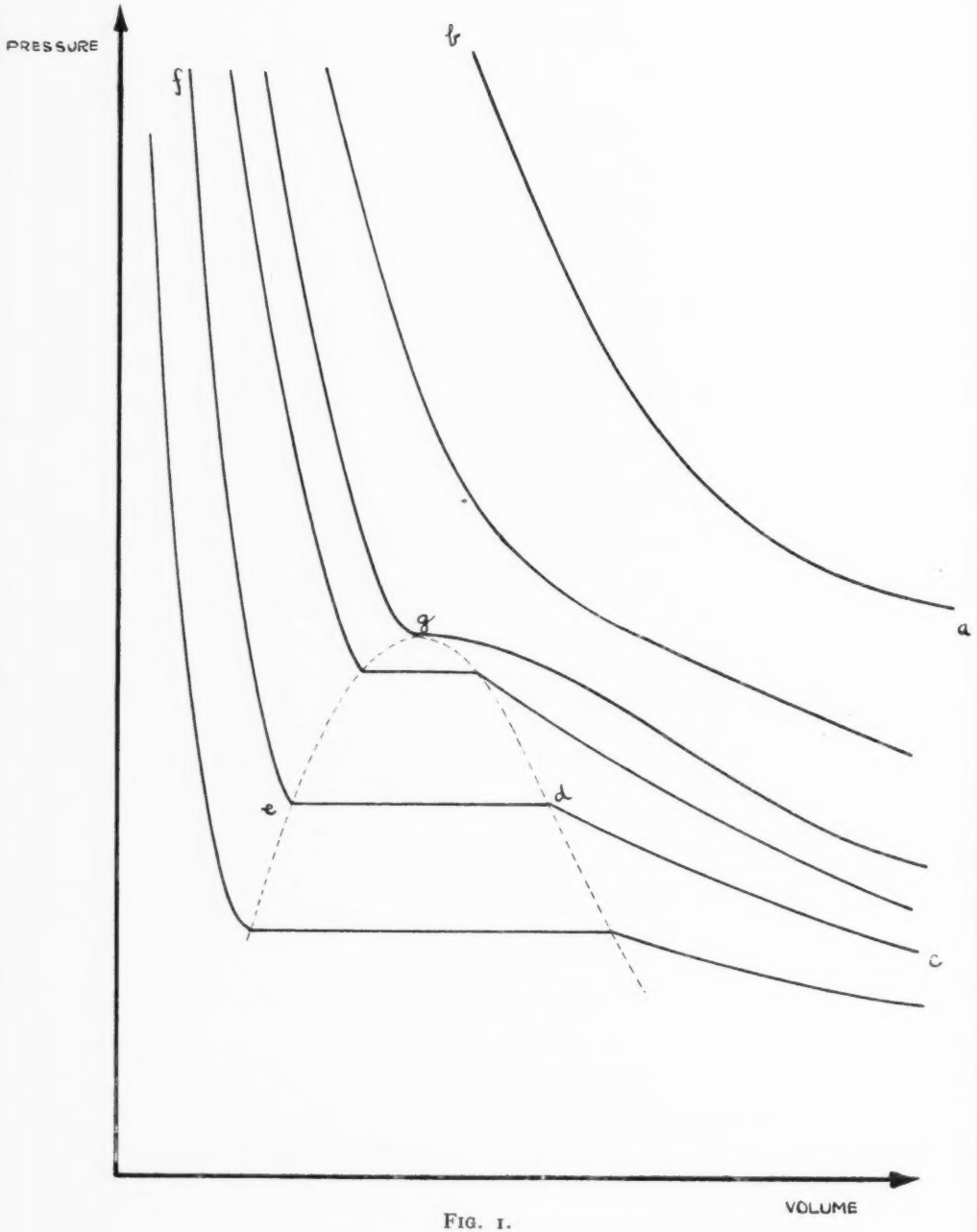

FIG. 1.

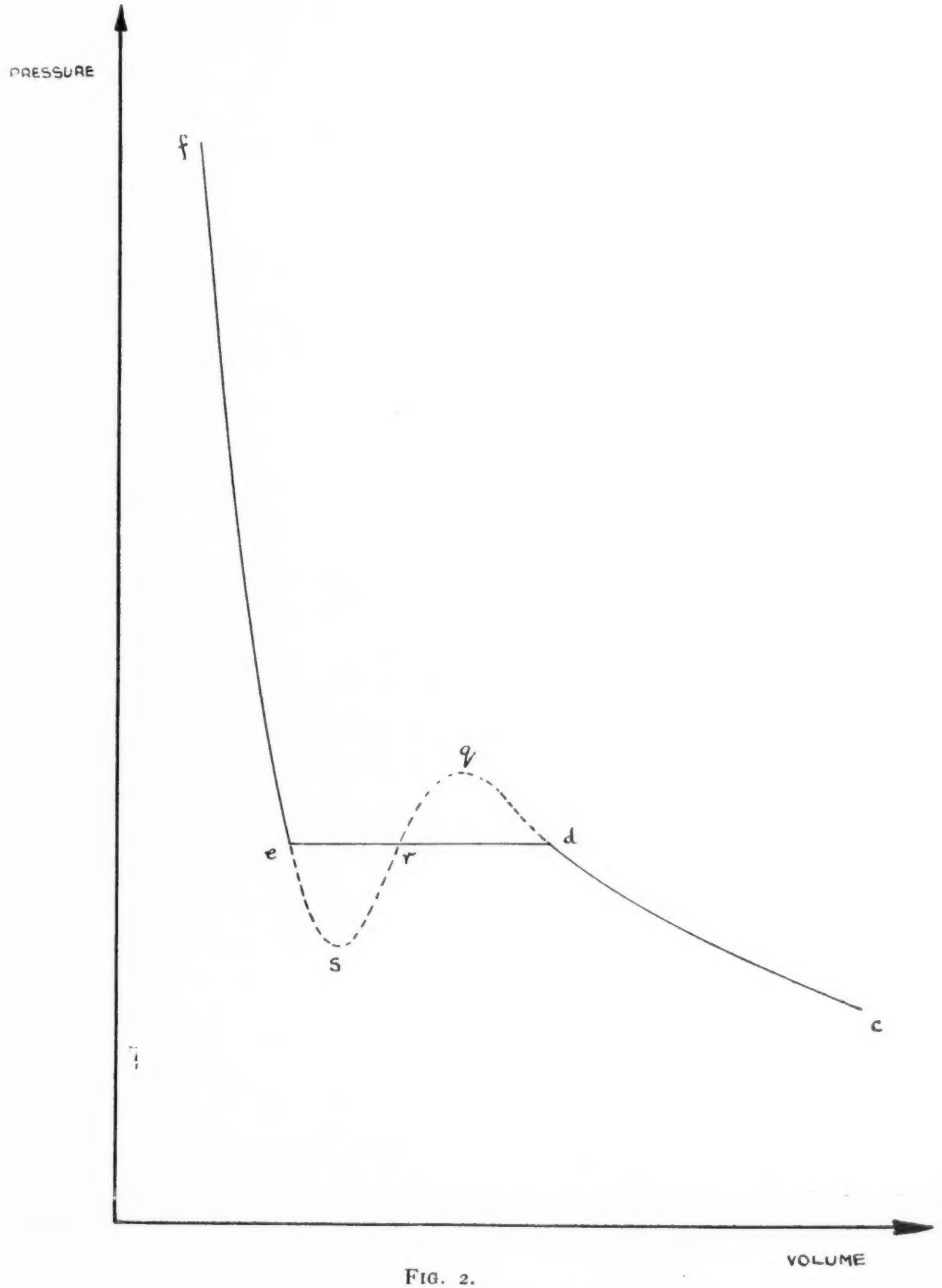

FIG. 2.

The suggestion that this might be the case was originally made by Professor James Thomson, and later Maxwell proved from energy considerations that the areas $d\ q\ r$ and $r\ s\ e$ must of necessity be equal.

Accepting this "ideal isothermal" it is now possible to evolve an equation which represents to a considerable degree of accuracy the relation between pressure, volume and temperature of a fluid when in both the liquid and gaseous states.

The most famous and probably the most logical of such equations is the one given by Van der Waal, which can be written

$$(p + a/v^2)\ (v - b) = RT \qquad \text{(iii)}$$

where p, v, R, and T have the same meaning as in equation (i). a is a constant, the term a/v^2 representing the effect on the pressure of molecular attractions resisting the expansion of the gas, and b is a constant which is not much different from the liquid volume of the fluid.

This equation is only very roughly true, as it happens, for water and its vapour, but, nevertheless, by making use of it as a better approximation than the "Perfect Gas" Equation, it is possible to determine how great is the error involved in utilising the latter in the case of the vapour.

From the equation (iii), it can be shown that if p_c and T_c are the critical pressure and temperature of the substance, then:

$$a = 27\ (RT_c)^2/64\ p_c,\ b = RT_c/8\ p_c \qquad \text{(iv)}$$

Now equation (iii) can be written, by a simple transposition:

$$(pv/RT)\ (1 + a/pv^2)\ (1 - b/v) = 1$$

or

$$(pv/RT)\ [1 + a(RT/pv)^2 p/(RT)^2]\ [1 - b(RT/pv)\ (p/RT)\ ] = 1$$

Inserting the values of a and b from (iii) we have

$$a\ (RT/pv)^2\ p/(RT)^2 = (27/64)\ (T_c/T)^2\ (p/p_c)\ (RT/pv)^2$$
$$b\ (RT/pv)\ p/RT = (\tfrac{1}{8})\ (T_c/T)\ (p/p_c)\ (RT/pv)$$

For convenience put $1/x = pv/RT$, $\alpha = (27/64)\ (T_c/T)^2 (p/p_c)$, $\beta = (\tfrac{1}{8})\ (T_c/T)\ (p/p_c)$ Then equation (iii) reduces to

$$(1/x)\ (1 + \alpha x^2)\ (1 - \beta x) = 1$$
$$\text{or} \quad f(x) = \alpha\beta x^3 - \alpha x^2 + (1 + \beta)x - 1 = 0 \qquad \text{(v)}$$

Take now the case of water-vapour in the atmosphere. The extremes of temperature of the latter are very roughly $-50°$C. and $50°$C., the average being usually taken as $15°$C. Water-vapour may exist therein to any degree of saturation, but clearly for a given temperature the quantities α and β are greatest when p is a maximum. Since in practice the vapour behaves according to the "true" curve $c\ d\ e\ f$ instead of the "ideal" one $c\ d\ q\ r\ s\ e\ f$, this maximum is at d, where the vapour is just saturated. Also, since the saturated vapour pressure of water increases with temperature roughly according to a square law, it may be assumed without further proof that the maximum values of α and β occur when T is a maximum also.

Taking then $T = 50°\text{C.} = 323°\text{A.}$ we find for saturation $p = 123\text{mb.} = 123/1013.2$ atmos.[2] Also for water $T_c = 365°\text{C.} = 638\text{A.}$, $p_c = 194.6$ atmos. Hence

$$\alpha = (27/64)\ (638/323)^2\ (123/1013.2 \times 194.6) = .00103$$
$$\beta = (\tfrac{1}{8})\ (638/323)\ (123/1013.2 \times 194.6) = .00015$$

Clearly for the ranges of temperature and pressure that we are considering α and β are very small, and their squares, products, and higher powers may be neglected in comparison with unity.

The root of the equation (v) in which we are interested corresponds to points along $c\ d$ in the "isothermal" (Fig. 2) for which $pv = RT$ approx. or $x = RT/pv = 1$. Hence, as is otherwise obvious, $x = 1$ is a good approximation to the root.

[2] Kaye and Laby's "Physical and Chemical Constants," p. 42.

A better value can be found easily by Newton's "Method of Approximation"[3] which gives

$$\begin{aligned} x &= 1 - Lt_{x=1}\,[\,f(x)/f'(x)\,] \\ &= 1 - Lt_{x=1}\,[\,\{\,\alpha\beta x^3 - \alpha x^2 + (1+\beta)\,x - 1\,\}\,/\,\{\,3\alpha\beta x^2 - 2\alpha x + (1+\beta)\,\}\,] \\ &= 1 - \{\,\alpha\beta - \alpha + (1+\beta) - 1\,\}\,/\,\{\,3\alpha\beta - 2\alpha + (1+\beta)\,\} \\ &= 1 + (\alpha - \beta) \text{ by the Binomial Theorem, and neglecting } \alpha\beta. \end{aligned}$$

Thus along $c\,d$ on the "isothermal," Van der Waal's Equation reduces to

$$x = \frac{RT}{pv} = [\,1 + (\alpha - \beta)\,]$$

or $pv = RT[\,1 - (\alpha - \beta)\,]$, again by the Binomial Theorem.

The percentage error in using the "Perfect Gas" Equation for unsaturated water-vapour is thus 100 $(\alpha - \beta)$, which at 50°C. $= 100 \times (.00103 - .00015) = .088$ or less than 1/10th per cent. This is entirely negligible from the point of view we are considering, taking into account the relatively small amounts in which water-vapour is present in the atmosphere and in the gas-containers of airships.

The above equation is thus obeyed nearly exactly under atmospheric conditions by all the important gases (save saturated water-vapour), which are involved in lift calculations, and is therefore assumed in the subsequent analysis. The exception mentioned is of relatively much smaller importance, but is, however, dealt with more conveniently in the next paragraph.[4]

2. *The Density of Gases and their Mixtures*

Returning to equation (ii) $pv = RT/M$, since v is now the volume of 1 gram of a gas $v\rho = 1$ or $v = 1/\rho$ where ρ is its density.

Hence, $p/\rho = RT/M$ or

$$\rho = pM/RT \qquad . \quad . \quad . \quad . \quad . \quad \text{(vi)}$$

For hydrogen, as we have seen, $M = 2.016$, and for helium, which has the molecular formula He and atomic weight 4, $M = 4$. Air is a mixture and, therefore, has to be dealt with in the manner specified at the beginning of paragraph 1. The International Commission for Air Navigation has laid down that its average composition (dry) can be taken as[5]:—

Constituent.	Percentage by volume.	Density at 0 C and pressure 1013 2 mb.
Nitrogen	78.03	1.2507
Oxygen	20.99	1.4290
Argon	0.94	1.7809
Carbon-dioxide	0.04	1.9769

Since the molecular weight of Nitrogen (N_2) is $2 \times 14.008 = 28.016$,
" " " Oxygen (O_2) is $2 \times 16 = 32$,
" " " Argon (A) is $1 \times 39.91 = 39.91$,
" " " CO_2 is $(1 \times 12) + (2 \times 16) = 44$,

we have, from the above rule,

$$M \text{ for air} = [\,(78.03 \times 28.016) + (20.99 \times 32) + (.94 \times 39.91) + (.04 \times 44)\,]/100 = 28.971$$

In the case of unsaturated water-vapour the molecular formula is H_2O, so that $M = (2 \times 1.008) + (1 \times 16) = 18.016$. At saturation, moreover, the density remains constant whatever the volume, *i.e.*, along the path $d\ e$ of Fig. 2, so that its value is the same as that at d, and equation (vi) applies in this case also, p being now the saturated vapour pressure at temperature T°A, complete tables

[3] See for example, Mellor's "Higher Mathematics for Students of Physics and Chemistry," p. 358.

[4] For a discussion on the effects of water-vapour on the pressure and temperature, etc., of the atmosphere, see also W. J. Humphreys' "Physics of the Air," Chap. V.

[5] Cf. Stewart's "Aircraft Instruments," p. 15.

of which exist.[6] Any other value of p corresponding to a point on $c\ d$ in the unsaturated state can be expressed in terms of the relative humidity, which is defined as the ratio, expressed in percentage, of the actual pressure of water-vapour in a given space to that which would exist for saturation at the same temperature. This quantity can be read directly from the " Hair Hygrometer " or calculated, with the aid of tables, from the readings of such an instrument as the " Wet and Dry " Hygrometer.

Strictly speaking, in a large volume of water-vapour at uniform temperature it is only possible for equilibrium to exist if the latter be unsaturated save at the lowest point. For since there is bound to be an increase of pressure, however small, as one proceeds downwards (see next paragraph), the value of the pressure at the lowest point cannot be greater than, but may be equal to, that which corresponds to the point d in Fig. 2. Any other condition is unstable, for if it so happened that the pressure at some higher point had acquired this latter value, then that in the lower layer would have to be greater, which is impossible without condensation occurring until the above condition were re-established. By Dalton's Law the same applies even though the water-vapour be mixed with other gases. In a sense, therefore, we are entitled to consider all large volumes of gases containing water-vapour as unsaturated at uniform temperature, even although the pressure differences are so small that no instrument would detect them, and in practice complete equilibrium is never obtained, due to convection currents.

This being the case, the rule already given in paragraph 1 for the determination of " average " molecular weights follows immediately. Imagine a certain volume v made up of a number of " compartments " each of volume $v_1\ v_2 \ldots v_n$ so that $v = v_1 + v_2 + \ldots + v_n$.

Let these compartments be filled with different pure gases at the same pressure p and temperature T°A., the molecular weights of these gases being $M_1\ M_2 \ldots M_n$. Then the densities of the latter are given by pM_1/RT, $pM_2/RT, \ldots pM_n/RT$ and their masses by pv_1M_1/RT, $pv_2M_2/RT, \ldots pv_nM_n/RT$. Let now the imaginary partitions be removed and the gases merged. In a short time the mixture will become uniform and each gas will have a partial pressure given by the " Perfect Gas " Equation, and so equal to $(v_1/v)p$, $(v_2/v)p, \ldots$ $(v_n/v)p$; this applies, even if one of the constituents were originally saturated water-vapour, since there has been an expansion in every case.

The pressure and temperature of the mixture is of course the same as before, also the total mass, and since all the constituents now obey the " Perfect Gas " Equation, so also does the mixture. Hence from the definition of the average M, and from the constancy of mass

$$pvM/RT = pv_1M_1/RT + pv_2M_2/RT + \ldots pv_nM_n/RT$$

Whence

$$M = (M_1v_1 + M_2v_2 + \ldots + M_nv_n)/v \qquad \text{(vii)}$$

and the formula is proved.

Again from the definition of relative humidity, and from equation (vii) we clearly have :—

$$\rho/\rho_s = p/\pi_T = r \qquad \text{(viii)}$$

where ρ = density of unsaturated vapour.

ρ_s = density of vapour for saturation at same temperature T°A.

p = pressure of vapour.

π_T = saturated vapour pressure at same temperature T°A.

r = relative humidity/100.

A practical rule[7] which is sometimes of value in calculating water-vapour

[6] *e.g.*, Glaisher's " Hygrometrical Tables " or Kaye and Laby's " Physical and Chemical Constants," p. 42.

[7] Cf. " Handbook of Lambrecht Polymeter," pp. 4-5.

density approximately is that the latter, measured in grams per cubic metre, is numerically equal to the vapour pressure measured in millimetres of mercury.

This can be easily proved as follows:—

Let the pressure p of the vapour be h millimetres of mercury. Then, since the density of the latter is approximately 13.6 grams per cm.^{3} and $g=981$ cm./sec.^{2}:—

$$p = (h/10) \text{ cm.} \times 13.6 \text{ gr./cm.}^3 \times 981 \text{ cm./sec.}^2 \quad (p = pgh).$$

Taking the average temperature of the atmosphere as 15°C. or 288°, we have from equation (vi)

$$\rho = (18.016 \text{ gr.} \times {}^\circ\text{A.})/(83.15 \times 10^6 \text{ ergs}) \times (h \text{ cm.}/10) \times (13.6 \text{ gr./cm.}^3) \times (981 \text{ cm./sec.}^2) \times (1/288{}^\circ\text{A.})$$
$$= (1.003h) \text{ (gr./10}^6 \text{ cm.}^3) = 1.003h \text{ grams per cubic metre.}$$

It is seen, therefore, that the rule is very nearly accurate at 15°C. At other temperatures there will be a discrepancy, but not a very serious one, since in the term for absolute temperature the added 273° is large compared with probable temperature variations.

Inserting the various values of M, we have now:—

$$\left.\begin{aligned} &\text{Density of dry air} = 28.971\ p/RT \\ &\quad,, \quad \text{hydrogen} = 2.016\ p/RT \\ &\quad,, \quad \text{helium} = 4\ p/RT \\ &\quad,, \quad \text{water-vapour} = 18.016\ r\pi_T/RT \end{aligned}\right\} \quad . \quad . \quad \text{(ix)}$$

Consider the case of moist air; isolate a volume and let, as before, p be the pressure, T°A. the temperature, r the relative humidity, and π_T the saturated vapour pressure of water at temperature T.

From Dalton's Law and equations (viii) and (ix)

$$\text{Partial pressure of dry air} = p - r\pi_T.$$
$$\text{Hence the density of the dry air alone} = 28.971\ (p - r\pi_T)/RT.$$
$$\text{Hence the density of the water-vapour} = 18.016\ r\pi_T/RT$$

from equation (viii).

Therefore, the density of the mixture is

$$\begin{aligned} &28.971\ (p - r\pi_T)/RT + 18.016\ r\pi_T/RT \\ &= (1/RT)\ (28.971\ p - 28.971\ r\pi_T + 18.016\ r\pi_T) \\ &= (28.971\ p/RT)\ \{\ 1 - (10.955/28.971)\ (r\pi_T/p)\ \} \\ &= (28.971\ p/RT)\ \{\ 1 - .378\ (r\pi_T/p)\ \} \quad . \quad . \quad . \quad \text{(x)} \end{aligned}$$

But, from equation (ix) the quantity $28.971p/RT$ is the density which the air would have if perfectly dry. Hence, moist air is lighter than dry and the term $(1 - .378\ r\pi_T/p)$ is a correction factor to be applied for humidity in the atmosphere.[8]

Next, consider a mixture of hydrogen, air, and water-vapour, such as is usually found in the interior of a gas-bag or other container when nominally filled with hydrogen. We shall here introduce a new term, the "dry purity," denoted by K; this is defined as the ratio of the partial pressure of the hydrogen to the combined partial pressures of the hydrogen and the air but *not* the water-vapour, which from Avogadro's Law is the same as the ratio of the number of hydrogen molecules to the total number of hydrogen and air. Hence what may be called the "percentage dry purity" is equal to $100K$; this is not quite, although nearly, the same as the "percentage purity" as usually defined in airship work,[9] but the former definition will be shown to be more systematic, and has certain decided practical advantages.

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[8] Cf. Kaye and Laby's "Physical and Chemical Constants," p. 23.

[9] Cf. Scott and Richmond, "The Effect of Meteorological Conditions on Airships," Appendix (III.). Journal R.Ae.S., March, 1924.

With the other symbols as before, we now have:—

Combined partial pressures of hydrogen and air $= p - r\pi_T$

Hence partial pressure of hydrogen $= K(p - r\pi_T)$

" " air $= (1-K)(p - r\pi_T)$

Density of hydrogen $= 2.016K(p - r\pi_T)/RT$

" air $= 28.971(1-K)(p - r\pi_T)/RT$

"water-vapour $= 18.016\, r\pi_T/RT$

$$\text{Density of mixture} = \frac{2.016K(p - r\pi_T)}{RT} + \frac{28.971(1-K)(p - r\pi_T)}{RT} + \frac{18.016\, r\pi_T}{RT}$$

$$= (1/RT)(2.016\,Kp - 2.016\,Kr\pi_T + 28.971\,p - 28.971\,r\pi_T - 28.971\,Kp + 28.971\,Kr\pi_T + 18.016\,r\pi_T)$$

or, density of impure and moist hydrogen

$$= [28.971\,p/RT]\,[1 - (26.955/28.971)\,K + (26.955/28.971)(Kr\pi_T/p) - (10.955/28.971)(r\pi_T/p)] \quad . \quad . \quad . \quad . \quad \text{(xi)}$$

The expression is left in this form for convenience in the subsequent analysis. The original form shows, as is otherwise obvious, that moist and impure hydrogen is very much heavier than the pure gas, so that, taking into consideration the comparative lightness of moist air relative to dry, we should expect, as is found in practice, a definite loss of lift due to humidity.

Similarly, if a mixture of helium, air and water-vapour be considered, it is found, by a simple change of constants, that:

Density of impure and moist helium

$$= [28.971\,p/RT]\,[1 - (24.971/28.971)\,K + (24.971/28.971)(Kr\pi_T/p) - (10.955/28.971)(r\pi_T/p)] \quad . \quad . \quad . \quad . \quad \text{(xii)}$$

As with hydrogen, moist and impure helium is thus heavier than the pure gas, so that in this case also we should expect a loss of lift due to humidity.

3. *The Equilibrium of a Column of Gas*

In considering the conditions of pressure, etc., in vertical columns of gases such as might be imagined in and around a gas-container, it is usual to make the following assumptions:—

(*a*) That the gases obey the "Density Equation" given by equation (vi) of the last paragraph. This, as has been demonstrated, is sensibly the case for all with which we are concerned, at atmospheric pressures and temperatures.

(*b*) That the effects of any vertical accelerations on the pressure can be neglected. This proviso also can be accepted in the present state of knowledge about their magnitude.

(*c*) That no horizontal temperature gradient exists in the gases. As regards the atmosphere, such gradients are very "flat," and normally quite inappreciable taken over the width of an airship. Such would not seem necessarily the case, however, with the gas in the interior of a gas-container. To take an example, suppose that an airship were flying for a long period in the early morning or late evening such that one side were continually exposed to strong solar radiation, and the other were sheltered. In that case one might expect quite an appreciable horizontal temperature gradient in the lifting-gas were it not for the effects of convection currents. As, however, such conditions are ideal for the generation of the latter, it can be assumed that any such gradient would be rapidly damped out. On the whole, therefore, this assumption is also justified.

(*d*) That the change of gravity with height is negligible. As is shown in any text-book on ballistics, this is the case for any altitude at which airships are ever likely to fly.

The distribution of pressure, etc., in such columns can thus be deduced from the conditions of static equilibrium in the usual manner as follows:—

Consider such an imaginary vertical column of square section passing through a mass of gas; the column is supposed to be of unit cross-sectional area, and is represented in Fig. 3. Take any horizontal plane as the plane of reference, such as, for example, ground level, and let the point O be at the centre of the cross-section of the column. Denote the pressure, density, and temperature of the gas at O by p_o ρ_o and T_o °A respectively, the corresponding values for a point P at height h being p ρ and T.

Now isolate a small "slice" by the intersection with the column of two horizontal planes, one through P at height h and the other at height $h+dh$, and consider the equilibrium of this.

Then we have, since the prism is of unit cross-sectional area:—

Force acting on top face of slice downwards $=p+dp$

Force acting on bottom face of slice upwards $=p$

Weight of slice $=\rho g dh$ acting downwards.

Equating all forces to zero for equilibrium

$$(p+dp)-p+\rho g dh=0$$

$$\text{or } dp=-\rho g dh.$$

But since the gas is assumed to obey the "Density equation" (vi) we have also

$$\rho=pM/RT.$$

Substituting in the above

$$dp=-(pM/RT)\,g dh$$

$$\text{or } dp/p=-(gM/R)\,(dh/T).$$

Integrating for all the similar slices between P and O we get:—

$$\log_e p+c=-(gM/R)\int_o^h dh/T \text{ where } c \text{ is some constant.}$$

But when $h=0$ clearly $\int_o^h dh/T=0$ and $p=p_o$ so that $c=-\log_e p_o$

$$\text{Hence } \log_e p-\log_e p_o=-(gM/R)\int_o^h dh/T$$

$$\text{or } \log_e (p/p_o)=-(gM/R)\int_o^h dh/T. \quad . \quad . \quad . \quad . \quad \text{(xiii)}$$

The integral $\int_o^h dh/T$ can be evaluated if the relation between T and h be known for all values of the latter, whence p can be determined relative to p_o. It is not generally practicable, however, to measure T directly except at a few fixed points, so that its value at others has to be estimated; fortunately, however, this is rarely a serious disability, since temperature occurs in the integral expression in absolute units (°A.) and quite a considerable error, even of several degrees, may be permitted without introducing a large percentage inaccuracy in the calculated value of p at any point.

For columns of gas in and around an airship gas-container the most justifiable assumption is that the temperature varies sensibly in a linear manner with height. This is certainly true of the atmosphere, where the existence of a general "lapse-rate" has long been known (paragraph 5). Even in cases of "inversion" of the temperature near the ground, it may be accepted that as the maximum diameter of an airship is not likely in the near future to exceed 200ft. the change of tem-

perature is also linear in the atmospheric zone immediately round the ship. The situation inside a gas-container, however, is somewhat different, as, due to the convection currents already noticed, there is a general tendency towards equalisation of temperature and " isothermal " conditions. The only circumstances in which a considerable alteration of temperature with height is likely to occur in this case is when the ship is exposed to strong solar radiation from directly overhead (*e.g.* at mid-day in the tropics) and also when it is housed in a shed and the roof of the latter has become heated by the sun; in the latter case, as is well known, there can often be quite a large difference of temperature between the floor and roof. Since the hotter gas is now at the top of the container, such a condition is quite stable, and should obviously be considered in any investigation into possible errors in existing lift formulæ.

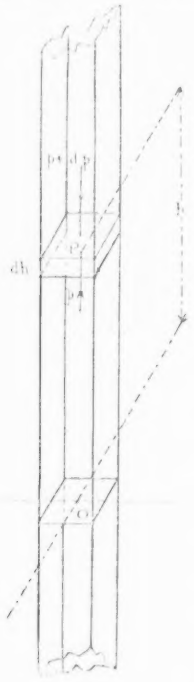


FIG. 3.

In the mathematical treatment, however, the " isothermal " assumption is obviously merely a particular case of the more general " linear " law.

If a " linear " law be accepted, then, we can write $T=T_o-\sigma h$ where σ is the rate of change of temperature with height, or " lapse rate," positive if the temperature decreases, negative if it increases. Substituting in equation (xiii) we now have:—

$$\log_e p/p_o = -(gM/R)\int_o^h dh/(T_o-\sigma h)$$

$$= (gM/R\sigma)\left[\log_e (T_o-\sigma h)\right]_o^h$$

$$= (gM/R\sigma)\log_e \{ (T_o-\sigma h)/T_o \}$$

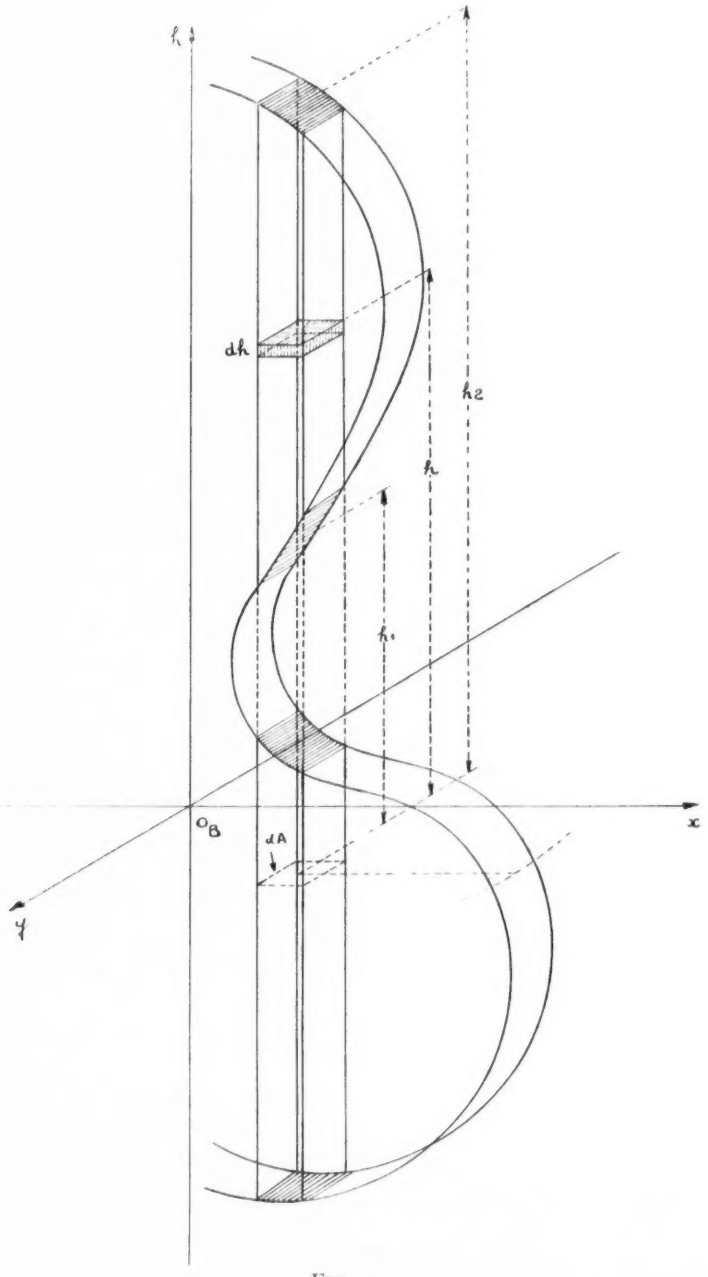


FIG. 4.

$$\left.\begin{aligned}\text{whence } p &= p_0 \{ (T_0 - \sigma h)/T_0 \}^{gM/R\sigma} \\ &= p_0 (1 - \sigma h/T_0)^{gM/R\sigma}\end{aligned}\right\} \quad . \quad . \quad . \quad \text{(xiv)}$$

Since in practice $\sigma h/T_0$ is very small compared with unity, the expression in brackets can be expanded by the Binomial Theorem giving:

$$p = p_0 [1 - (gM/R\sigma)(\sigma h/T_0) + \{ (gM/R\sigma)(gM/R\sigma - 1)/2! \} (\sigma h/T_0)^2 - \{ (gM/R\sigma)(gM/R\sigma - 1)(gM/R\sigma - 2)/3! \} (\sigma h/T_0)^3 + \ldots \text{etc.}]$$

which can be written, by an obvious modification:

$$p = p_0 [1 - \{ (M/RT_0) gh \} + \{ (1 - R\sigma/gM)/2! \} ([M/RT_0] gh)^2 - \{ (1 - R\sigma/gM)(1 - 2R\sigma/gM)/3! \} ([M/RT_0] gh)^3 + \ldots \text{etc.}]$$

The term $(M/RT_0) gh$ varies directly as the height h and inversely as the temperature T_0 at the plane of reference; in practice the former would not be less than -55°C. or 218°A.

In a subsequent paragraph it will be found of some importance to determine the percentage error involved in measuring the pressure of the atmosphere at the control-car of an airship instead of at the height of the C.B. of a gas-container, the latter value being, strictly speaking, required in lift calculations. If we consider a ship of maximum cross-sectional diameter 60 metres or, say, 200ft. (which is much greater than any existing or projected), and suppose the "plane of reference" to be the horizontal one passing through the manometer at the control-car, then since this C.B. is roughly along the central axis h may be assumed to be not much greater than 30 metres when the ship is horizontal in flight. Hence, with the lowest temperature practically encountered

$$(M/RT_0) gh = \{ (28.971 \text{ gr.} \times \text{°A.})/(83.15 \times 10^6 \text{ ergs}) \} \times (1/218\text{°A.}) \times (981 \text{ cm./sec.}^2) \times (30 \times 10^2 \text{ cms.})$$

$$= .0047 \text{ approx., the units cancelling out.}$$

The third term in the expansion tends to become less (and therefore the total error to be increased) the greater the positive value of σ. A maximum for the latter may be taken as that corresponding to the "adiabatic" of dry air, which is 9.7°C. per 1,000 metres. This value makes

$$R\sigma/gM = (83.15 \times 10^6 \text{ ergs}/28.971 \text{ gr.} \times \text{°A.}) \times (9.7\text{°A.}/10^5 \text{ cm.}) \times (\text{sec.}^2/981 \text{ cm.})$$

$$= .3 \text{ approx., the units again cancelling out.}$$

In this case, then, equation (xv) becomes

$$p = p_0 [1 - .0047 + \{ (1 - .3)/2! \} (.0047)^2 - \{ (1 - .3)(1 - .6)/3! \} (.0047)^3 \ldots \text{etc.}]$$

and the error involved is less than $\frac{1}{2}$ per cent.

Since air is the densest gas with which we are practically concerned, the above example is sufficient to prove that in the expansion of the expression for p the terms become extremely convergent after the first two.

The corresponding error involved in measuring the absolute temperature of the atmosphere at the control-car instead of at the C.B. can be easily found from the expression $T = T_0 - \sigma h = T_0 (1 - \sigma h/T_0)$

Putting, as before, $\sigma = 9.7$°C. per 1,000 metres, $h = 30$ metres, $T_0 = 218$°A.

$$\sigma h/T_0 = (9.7\text{°A.}/1{,}000 \text{ metres}) \times (30 \text{ metres}/218\text{°A.}) = .001 \text{ approx.}$$

and the error is 1/10th per cent.

The latter is not, however, the maximum error which can occur in this case, for σ may have very much larger values numerically, although of opposite sign. In the case of a shed heated in the manner already suggested, we easily might find a difference of 3°C. between the roof and floor. Taking the height of the shed as 150ft. this gives $\sigma = -(3\text{°A.}/150\text{ft.})$ and $h = 75$ft. The temperature T_0 at the ground would in this case be about normal, or, say, equal to 15°C. or 288°A. Hence:

$$\sigma h/T_0 = -(3\text{°A.}/150 \text{ ft.}) \times (75 \text{ ft.}/288\text{°A.}) = -.005 \text{ approx.,}$$

and the error involved is about $\frac{1}{2}$ per cent.

Lastly, equation (xiv) can be written in the form:—

$$p = p_0 - (p_0 M/RT_0)\, gh + \{ (1 - R\sigma/gM)/2! \} \cdot \{ (p_0 M/RT_0)\, gh \}^2/p_0$$
$$- \{ (1 - R\sigma/gM)\,(1 - 2\,R\sigma/gM)/3! \}\ \{ (p_0 M/RT_0)\, gh \}^3/p_0^2 \ldots \text{etc.}$$

or, since $\rho_0 = p_0 M/RT_0$

$$p = p_0 - \rho_0 gh + \{ (1 - R\sigma/gM)/2! \}\ \{ (\rho_0 gh)^2/p_0 \}$$
$$- \{ (1 - R\sigma/gM)\,(1 - 2\,R\sigma/gM)/3! \}\ \{ (\rho_0 gh)^3/p_0^2 \} + \ldots \text{etc.} \quad . \qquad \text{(xv)}$$

which shows the deviation of the " true " value of p from that given in text books on hydrostatics for an incompressible fluid. This deviation is evidently extremely minute for the comparatively small differences of height with which we are concerned in the present connection.

4. *The Law of Displacement*

We are now in a position to examine the well-known equation which is deduced from Archimedes' " Law of Displacement " and which is the foundation of all lift formulæ. This law states that in a state of equilibrium a body immersed in a fluid experiences an upward force through its centre of buoyancy equal in magnitude to the differences between its weight and that of the fluid it displaces. In this form the law is perfectly exact and applicable to both compressible and (comparatively) incompressible fluids; that such is the case can be easily seen from à priori reasoning. For the pressures exerted by the surrounding fluid at points on the surface of the body are obviously not changed if for the latter an identically equivalent volume of fluid be substituted. But since in this case the latter is now uniform, the sum of the upward components of the forces produced by these pressures, each acting on their adjacent small areas, must generate a resultant force which is the same in magnitude and point of application, but opposite in direction, to the weight of the substituted fluid. From this the above statement of the law follows.

In the case of a gas-container, then, to determine the lift it is necessary to find an expression for the weight of gas that it contains, and for that of the air which it displaces. This may be effected in the following manner:

Suppose the container to be of any shape whatsoever, and take as axes $O_B x$, $O_B y$, $O_B h$ (the latter vertical) through O_B, the C.B. of its total volume V (Fig. 4). Divide the sectional area in which the plane $O_B xy$ cuts the container into small squares, each of area dA, by lines parallel to $O_B x$ and $O_B y$ respectively; take any one of these squares and erect thereon an imaginary vertical column similar to that of the last paragraph. This will cut the sides of the container in an even number of places, generally two (four are shown in the figure), such that the column alternately passes inside and outside the volume. Suppose h_1 and h_2 be the height of two such consecutive places above the $O_B xy$ plane (h_2 being the greater than h_1, due regard being given to sign) and that the column between the two is inside the container.

The weight of any " slice," either of lifting gas or displaced air, in this part of the column between the heights h and $h + dh$ will be equal to $\rho g \,.\, dA \,.\, dh$ which also equals $(pM/RT)\, g \,.\, dA \,.\, dh$ where p, ρ, and $T°$A. are as usual the pressure, density and temperature of the lifting gas or displaced air (whichever is for the moment being considered) at height h above the " plane of reference " $O_B xy$; the other terms have also their usual significances.

But from equation (xiv) $p = p_0 (1 - \sigma h/T_0)^{gM/R\sigma}$, where p_0 and T_0 are now the pressure and temperature at the latter plane. Also $T = T_0 - \sigma h$, so that

$$\text{Weight of " slice "} = [\ \{ p_0 (1 - \sigma h/T_0)^{gM/R\sigma}\, M \} / \{ RT_0 (1 - \sigma h/T_0) \}\ ]\, g \,.\, dA \,.\, dh$$
$$= (gM/RT_0)\ dA\ [\, p_0 (1 - \sigma h/T_0)^{(gM/R\sigma - 1)}\, dh\,]$$

Hence the total weight of the gas (or displaced air) in the part of the column between h_1 and $h_2 = \Delta W$

$$= (gM/RT_o)\, dA \int_{h_1}^{h_2} p_o\,(1 - \sigma h/T_o)(gM/R\sigma - 1)\, dh$$

$$= -(gM/RT_o)\, dA\, [p_o\,(1 - \sigma h/T_o)^{gM/R\sigma}/(gM/R\sigma)\,(\sigma/T_o)]_{h_1}^{h_2}$$

$$= -dA\, [p]_{h_1}^{h_2} \text{ since } p = p_o\,(1 - \sigma h/T_o)^{gM/R\sigma}$$

$= (p_1 - p_2)\, dA$ where p_1 and p_2 are the pressures at heights h_1 and h_2.

Substituting the alternative expression for p_1 and p_2 given by equation (xv) of the last paragraph, and carrying the expansion to the fourth term only, we get

$$\begin{aligned}\Delta w = [\, & p_o - \rho_o g h_1 + \{(1 - R\sigma/gM)/2!\}\ \{(\rho_o g h_1)^2/p_o\} \\ & - \{(1 - R\sigma/gM)(1 - 2R\sigma/gM)/3!\}\ \{(\rho_o g h_1)^3/p_o^2\} \\ & - p_o + \rho_o g h_2 - \{(1 - R\sigma/gM)/2!\}\ \{(\rho_o g h_2)^2/p_o\} \\ & + \{(1 - R\sigma/gM)(1 - 2R\sigma/gM)/3!\}\ \{(\rho_o g h_2)^3/p_o^2\}\,]\, dA \\ = [\, & \rho_o g\,(h_2 - h_1) - L\,(h_2^2 - h_1^2) + N\,(h_2^3 - h_1^3)]\, dA\end{aligned}$$

where for convenience

$$\{(1 - R\sigma/gM)/2!\}\ \{(\rho_o g)^2/p_o\}$$

is put equal to L, and

$$\{(1 - R\sigma/gM)(1 - 2R\sigma/gM)/3!\}\ \{(\rho_o g)^3/p_o^2\}$$

to N. Hence

$$\begin{aligned}\Delta w &= (h_2 - h_1)\, dA\, [\rho_o g - L\,(h_1 + h_2) + N\,(h_1^2 + h_1 h_2 + h_2^2)] \\ &= \Delta V\, [\rho_o g - L\,(h_1 + h_2) + N\,(h_1^2 + h_1 h_2 + h_2^2)]\end{aligned}$$

where ΔV denotes the volume of the column between the heights h_1 and h_2.

$$= \rho_o g \Delta V - L\,(h_1 + h_2)\,\Delta V + N\,(h_1^2 + h_1 h_2 + h_2^2)\,\Delta V.$$

If now we sum up for all the similar parts of the columns all over the total volume V we get:—

Weight of lifting gas (or displaced air) W

$$= \rho_o g V - L\Sigma\,(h_1 + h_2)\,\Delta V + N\Sigma\,(h_1^2 + h_1 h_2 + h_2^2)\,\Delta V$$

where the Σ denotes summation as indicated.

But $(h_1 + h_2)/2$ is the distance of the C.B. of the small volume ΔV from the plane $O_B xy$ and therefore $\frac{1}{2} \times (h_1 + h_2) \times \Delta V$ is the moment of ΔV about this plane. The quantity $\Sigma\,(h_1 + h_2)\, dV$ is thus twice the sum of all such moments taken over the whole volume V, which equals zero since O_B is, by hypothesis, at the C.B. of the latter.

The above equation thus reduces to

$$\rho_o g V\,[1 + N\Sigma\,(h_1^2 + h_1 h_2 + h_2^2)\,\Delta V/\rho_o g V]$$

and the quantity

$$(N/\rho_o g)\,[\Sigma\,(h_1^2 + h_1 h_2 + h_2^2)\,\Delta V/V] \times 100$$

is the percentage error involved in assuming the mass of gas (or displaced air) to be equal to the volume of the container multiplied by the density of the gas considered at the level of the former's C.B. It can easily be demonstrated that this error is exceedingly minute.

For since in practice σ for either the atmosphere or for a lifting gas ranges from about $+0.7$°C. per km. to -70°C. per km. (a possible temperature gradient inside a shed) the quantity $R\sigma/gM$ ranges accordingly from $+.3$ to -2 and $(1 - R\sigma/gM)(1 - 2R\sigma/gM)/3!$ from .05 to 2.5.

For a spherical gas bag or container $V = (4/3)\,\pi r^3$ it can be shown that for this figure $\Sigma\,(h_1^2 + h_1 h_2 + h_2^2)\,\Delta V$ equals $(4/5)\,\pi r^5$. Substituting, we find that the term for the error becomes

$$100\ \{(1-R\sigma/gM)(1-2R\sigma/gM)/3!\}\ \{(\rho_0 g)^3/p_0^2\}$$
$$(1/\rho_0 g)\ \{(4\pi r^5/5)/(4\pi r^3/3)\}$$
$$=60\times\{(1-R\sigma/gM)(1-2R\sigma/gM)/3!\}(\rho_0 gr/p_0)^2$$

Taking the container to have a diameter of at most 200ft. this gives $r=100$ft. and $(\rho_0 gr/p_0)$ works out as in the last paragraph to have a maximum value of about .005 for air. Hence, putting the expression

$$(1-R\sigma/gM)(1-2R\sigma/gM)/L^2=2.5$$

we find the maximum percentage error

$$=60\times 2.5\times(.005)^2=.003.$$

This obviously is completely negligible, so that the equation $W=\rho_0 gV$ holds almost exactly with the stipulation already made as regards the value of ρ_0.

Hence, if ρ_A denote the density of the surrounding air at the level of the C.B. of a gas container, and ρ_G that of the gas therein at the same level, then the "Law of Displacement" gives:—

$$\text{Lift of container}=\rho_A gV-\rho_G gV$$
$$=gV(\rho_A-\rho_G) \quad . \quad . \quad . \quad . \quad . \quad \text{(xvi)}$$

This equation, which is fundamental in all lift formulæ, is of the same form as that usually deduced from the above law for "incompressible" fluids. We have thus demonstrated that it is of equally valid application (with negligible inaccuracy) to compressible ones, provided that all densities are measured in the appropriate plane specified.

5. *The Standard Atmosphere*

In calculations on the static lift of airships it is found convenient to take as a basis some "Standard Atmosphere," presumed to represent an average of meteorological conditions on the earth's surface. Unfortunately there has been in the past considerable lack of uniformity and agreement on such a standard, and from time to time the various authorities and organisations concerned with airship design have adopted different ones.

In the present analysis it is proposed to use the "Standard Atmosphere" accepted by the International Commission for Air Navigation (I.C.A.N.). There are several advantages inherent in so doing, the chief one being that this has been accepted since 1925 by all the principal countries engaged in aircraft production, including Great Britain (*vide* Air Publication 1173) as the basis for the calibration of aircraft instruments, especially those used in testing performance. It is to be expected, therefore, that in the near future practically all instruments in use will register according to this Convention; also, having been approved internationally, there is not likely to be any radical change for some time.

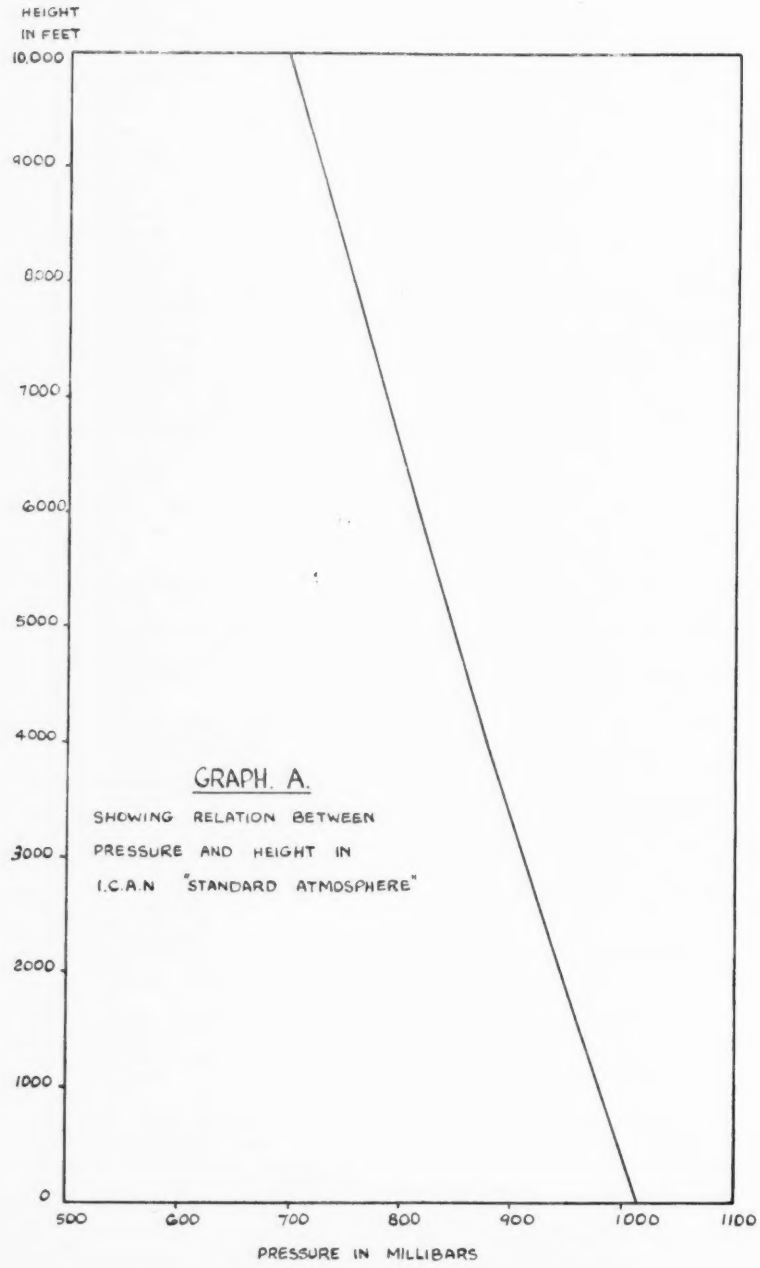
The Convention stipulates that in an I.C.A.N. "Standard Atmosphere":—

(i) The air is assumed dry and its chemical composition the same at all altitudes, this composition being that already given in paragraph 2. The value of g is taken uniformly as 980.62 cm./sec.^{2}.

(ii) The temperature at mean sea-level is assumed to be 15°C. or 59°F., and the barometric height 1013.2 millibars or 760 mm. of mercury at 0°C.

(iii) The temperature of the atmosphere between mean sea-level and a height of 11,000 metres or 36,090ft. is assumed to decrease uniformly at a rate of 6.5°C. per 1,000 metres, or 1.9812°C. per 1,000ft. (the "lapse rate"). This zone is called the "Troposphere."

(iv) The temperature of the atmosphere above 11,000 feet is assumed to be independent of altitude and equal to −56.5°C. This zone is called the "Stratosphere."



GRAPH. A.

SHOWING RELATION BETWEEN PRESSURE AND HEIGHT IN I.C.A.N "STANDARD ATMOSPHERE"

In airship work we are only concerned with temperature conditions as defined by (iii), and since in the "Troposphere" the temperature decreases in a "linear" law with height, equation (xiv) of paragraph 3 applies, the "plane of reference" now being mean sea-level. Corresponding to $\sigma = +6.5$°C. per km. and $M = 28.971$, we find $gM/R\sigma = +5.256$ so that in the I.C.A.N. "Standard Atmosphere" at height h

$$p = 1013.2 \left[\left\{ 288 - 1.9812\ (h/1000\text{ft.}) \right\} / 288 \right]^{5.256} \text{ mb.}$$
$$= 1013.2 \left\{ 1 - .006879\ (h/1000\text{ft.}) \right\}^{5.256} \text{ mb.} \quad . \quad . \quad \text{(xvii)}$$

a relation which is shown in Graph A. From the latter or the above equation it is therefore a simple matter to find the atmospheric pressure corresponding to any height registered by an "I.C.A.N." altimeter. It must be remembered, however, that neither graph nor formula applies in the case of an altimeter with adjustable zero, some instruments being so fitted in order that the dial will indicate directly height above point of departure, and also in order that the readings may be automatically corrected for a different pressure at mean sea-level than 1013.2 mb. As an alternative to the latter, however, the indications of an I.C.A.N. altimeter can be easily treated to give "true" height by the use of certain formulæ or "Computers."[10]

Similarly, the temperature at height h in the I.C.A.N. "Standard Atmosphere" is given by

$$T = \left\{ 288 - 1.9812\ (h/1000\text{ft.}) \right\} \text{°A.}$$
$$= 288 \left\{ 1 - .006879\ (h/1000\text{ft.}) \right\} \text{°A.} \quad . \quad . \quad . \quad \text{(xviii)}$$

a relation which is shown in Graph B.

The "lapse-rate" assumed in the I.C.A.N. "Standard Atmosphere" is a very good approximation indeed to the "true" value observed, for all seasons and latitudes, at heights above approximately 10,000ft. Below this altitude there is frequent instability and even "inversion" of the temperature gradient; also, of course, ground temperatures and pressures may be very different from those assumed in the Convention. Nevertheless, as a basis for the calibration of instruments, the latter could hardly be improved upon.

6. *The Lift of a Gas-Container and Influencing Factors*

The fundamental equations (x), (xi), (xii) and (xvi) already established enable us to deduce a practical formula for calculation of the lift of any gas container and hence of the static lift of an airship. Taking a perfectly general case, let:—

T°A. = the temperature of the atmosphere at the level of the C.B. of the container.

$(T + t)$°A. = the temperature of the lifting gas at the same level, so that t°C. is the "super-heating" of the gas (or "super-cooling" if negative in sign).

P = the pressure of the atmosphere at the level of the C.B.

$(P + p)$ = the pressure of the gas at the same level, so that p is the "super-pressure" thereat.

$100\ r_A$ = the relative humidity of the atmosphere (in %).

$100\ r_G$ = the relative humidity of the lifting gas (in %).

π_T = the saturated vapour pressure of water at temperature T°A.

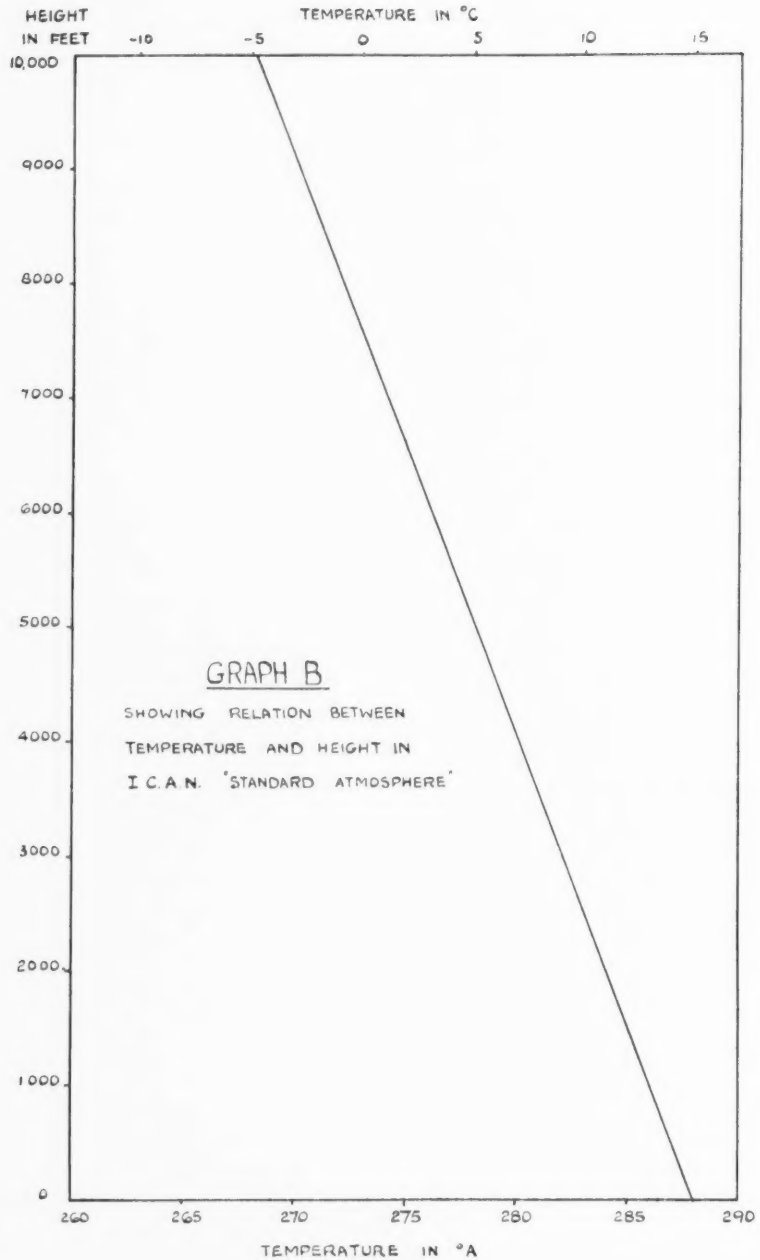
π_{T+t} = the saturated vapour pressure of water at temperature $(T + t)$°A.

$100\ K$ = the "percentage dry purity" of the lifting gas (paragraph 2).

V = the volume of the container.

Firstly, suppose the lifting gas to be hydrogen. Then, making the appropriate substitution in equations (x) and (xi), we have

[10] See C. J. Stewart's "Aircraft Instruments," Chap. I.


HEIGHT IN FEET
TEMPERATURE IN °C
-10
-5
0
5
10
15
10,000
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
260
265
270
275
280
285
290
TEMPERATURE IN °A
GRAPH B
SHOWING RELATION BETWEEN TEMPERATURE AND HEIGHT IN I.C.A.N. 'STANDARD ATMOSPHERE'

Density of atmosphere at level of C.B. $=\rho_A$

$$=(28.971\ P/RT)\ \{\ 1-(10.955/28.971)\ (r_A\pi_T/P)\ \}$$

Density of lifting gas at same level $=\rho_G$

$$=\{\ 28.971\ (P+p)/R\ (T+t)\ \}\ \{\ 1-(26.955/28.971)\ K + (26.955/28.971)\ [(Kr_G\pi_{T+t})/(P+p)] - (10.955/28.971\ (r_G\pi_{T+t})/[P+p])\ \}$$

Hence from equation (xvi)

$$\text{Gross lift of container}=gV\ (\rho_A-\rho_G)$$

$$=gV\ [(28.971\ P/RT)\ \{\ 1-(10.955/28.971)\ (r_A\pi_T)/P)\ \} - \{\ 28.971\ (P+p)/R\ (T+t)\ \}\ \{\ 1-(26.955/28.971)\ K + (26.955/28.971)\ (Kr_A\pi_{T+t}/\{\ P+p\ \}) - (10.955/28.971)\ (r_G\pi_{T+t}/[P+p])\ \}\ ]$$

$$=gV\ [28.971\ (P+p)/R\ (T+t)\ ] [(\{\ 1-(10.955/28.971)\ (r_A\pi_T/P)\ \}\ \{\ 1+t/T\ \})/(1+p/P) - \{\ 1-(26.955/28.971)\ K + (26.955/28.971)\ (Kr_G\pi_{T+t}/[P+p]) - (10.955/28.971)\ (r_G\pi_{T+t}/[P+p])\ \}\ ] \quad \text{(xix)}$$

by an obvious transposition.

This is, within the limits already specified, an accurate formula for calculating lift; it is, however, clearly unpractical in the above form. Now the circumstances when it is necessary to make such calculations on the gas containers of airships are roughly (*a*) during a " Lift-and-Trim " trial, (*b*) during normal flight. In the first case it is desirable to estimate the lift with all the accuracy at one's command, but, on the other hand, a day would naturally be chosen for the trial when atmospheric conditions were not abnormal. In the second case, abnormal conditions might occur, but it is not generally necessary to know the lift to such a degree of exactitude, a more important point being that the necessary calculations can be made expeditiously. For both cases, then, we are justified in examining the above formula and rejecting certain quantities which are small in comparison with others, always bearing in mind that the final total percentage error should not be greater than, say, $\frac{1}{2}$ per cent.

Such quantities which are small in comparison with unity are the non-dimensional ones p/P, t/T, $(10.955/28.971)\ (r_A\pi_T/P)$, $(10.955/28.971)\ (r_G\pi_T/\{\ P+p\ \})$ and $(1-K)$, and, as will be seen, we shall certainly be justified in rejecting their squares, higher powers, and products. Considering each term separately we find that :—

(i) p is always very small relative to P. Even in non-rigid and semi-rigid airships, where comparatively high " super-pressures " are found, p is rarely greater than 1in. of water or 2.5 mb.[11] Suppose " pressure-height " be at 30,000ft., which of course is at a greater altitude than even military airships are ever likely to operate, then P in this case $=300$ mb. and $p/P=2.5/300=.008$ approx. Clearly we can generally neglect the latter term altogether, and in any case put $1/(1+p/P)=1-p/P$, using the Binomial Theorem.

(ii) Scott and Richmond have shown[12] that with a 5,000,000 cu. ft. ship in stagnant air at 2000 metres the maximum superheating likely to occur in England is about 17°C., and will be less for a larger ship. Assuming the ground temperature to be 0°C. and a " lapse rate " of 0.5°C. per 1000 metres, the temperature at the above height will be -13°C. or 260°A. Hence $t/T=17/260=.065$.

Few figures are extant for the corresponding effect in tropical climates. Suppose, however, we take a ground temperature of 45°C.; with the same " lapse-rate " the temperature at 2000 metres will be 32°C. or 305°A. Assuming t to have a maximum value of 25°C., which is probably an over-estimate, then $t/T=25/305=.082$. In both cases an " inversion " of atmospheric temperature gradient would obviously tend to reduce this ratio.

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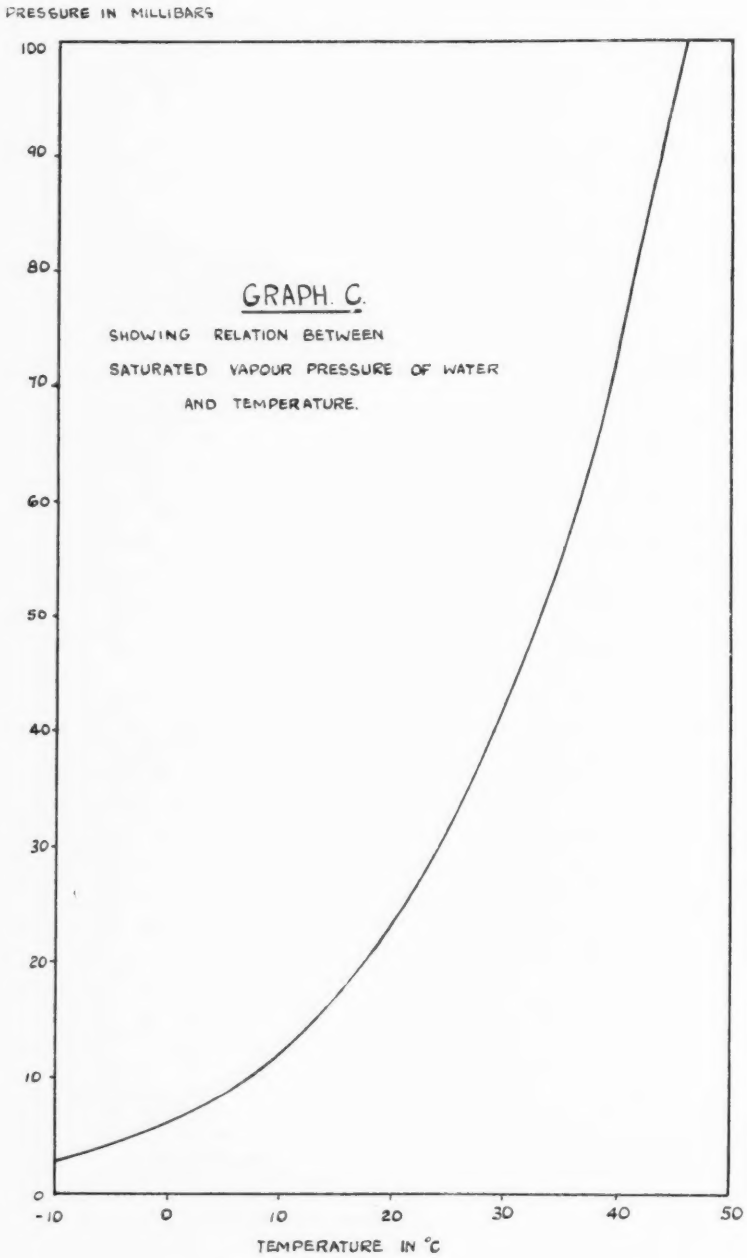
[11] Cf. Blakemore and Pagon's " Pressure Airships."

[12] " The Effect of Meteorological Conditions on Airships," Journal of R.Ae.S., March, 1924.


PRESSURE IN MILLIBARS
100
90
80
70
60
50
40
30
20
10
0
GRAPH. C.
SHOWING RELATION BETWEEN
SATURATED VAPOUR PRESSURE OF WATER
AND TEMPERATURE.
-10
0
10
20
30
40
50
TEMPERATURE IN °C

On the whole superheating is greater at higher altitudes because the solar radiation is then less absorbed by clouds, etc. On the other hand, an airship will in general be moving at considerable speed, and the cooling effect of this will tend to reduce any superheating in the gas. Consequently the latter value of t/T may be looked upon as a maximum.

(iii) r_A and r_G cannot be greater than unity, and the terms $(r_A\pi_T/P)$ and $r_G\pi_{T+t}/(P+p)$ are clearly a maximum, other things being equal, at saturation. The highest temperature encountered in the atmosphere is about 50°C., and this under conditions not usually associated with great humidity. For saturation at that temperature, however, we find $r_A\pi_T = 123$ mb., and assuming these figures to be for ground level, where $P = 1000$ mb. approx., this makes

$$\{ (10.955/28.971)/(r_A\pi_T/P) \} = .05 \text{ approx.};$$

similarly for $(10\,955/28.971)/r_G\pi_{T+t}/(P+p)$, since p is negligible in comparison with P.

At higher altitudes P will be less, but the temperature of the air also in general decreases, and with it, at a much greater rate, the saturated vapour-pressure at temperature T (Graph C). Hence, as we should not expect to find a temperature as great as 50°C. anywhere in the upper atmosphere, the above values may be accepted as a maximum.

(iv) The " percentage dry purity " $100\,K$ will be shown in a subsequent paragraph to be very nearly the same as the " percentage purity " as commonly defined. In actual practice the latter is rarely less than 95 per cent. and a value of 92 per cent. would scarcely be tolerated. Hence the maximum of $(1-K)$ may be taken as .08.

If we then reject all squares, products and higher powers of the above quantities in comparison with unity, and put $r_G\pi_{T+t}/(P+p) = r_G\pi_{T+t}/P$, equation (xix) becomes, by application of the Binomial Theorem to the first terms in the second square bracket:—

Gross lift of container

$$\begin{aligned}
&= gV\,[28.971\,(P+p)/R\,(T+t)]\,[1-(10.955/28.971)\,(r_A\pi_T/P)+t/T-p/P-1\\
&\qquad +(26.955/28.971)\,K-(26.955/28.971)\,(Kr_G\pi_{T+t}/P)\\
&\qquad +(10.955/28.971)\,(r_G\pi_{T+t}/P)]\\
&= gV\,[26.955\,K\,(P+p)/R\,(T+t)]\,[1+(28.971/26.955\,K)\,(t/T)\\
&\quad -(28.971/26.955\,K)\,(p/P)-(10.955/26.955\,K)\,(r_A\pi_{T+t}/P)-(r_G/\pi_{T+t}/P)\\
&\qquad +(10.955/26.955\,K)\,(r_G\pi_{T+t}/P)]
\end{aligned}$$

Removing $26.955\,K/28.971$ as a factor outside the second square bracket

$$\begin{aligned}
&= (26.955\;gVKP/RT)\,[\,\{\,1+(28.971/26.955\,K-1)\,(t/T)\,\}\\
&\qquad \{\,1-(28.971/26.955\,K-1)\,(p/P)\,\}\\
&\times\,\{\,1-(10.955/26.955\,K)\,(r_A\pi_T/P)-(1-10.955/26.955\,K)\,(r_G\pi_{T+t}/P)\,\}\,]
\end{aligned}$$

But since K is very nearly equal to unity, the term

$$(28.971/26.955\,K-1)$$

is approximately equal to .07, so that the maximum value of

$$1+(28.971/26.955\,K-1)\,(t/T)$$

is about $1+.07\times.08$ or 1.00056. The latter is thus negligibly different from unity and can be ignored. Similarly for the factor

$$\{\,1-(28.971/26.955\,K-1)\,(p/P)\,\}.$$

Again, the term $(10.955/26.955\,K)$ multiplies the small quantity $(r_A\pi_T/P)$, so that we can with sufficient accuracy assume a fair average of the " percentage dry purity " $100\,K$ as 96 per cent. and make the said term constant and equal to .42. For a similar reason $(1-10.955/26.955\,K)$ may be made constant and equal to .58.

The lift equation now becomes, therefore:—Gross lift of container filled with hydrogen

$$= [26.955\;gVKP/RT]\,[1-(.42\;r_A\pi_T+.58\;r_G\pi_{T+t})/P] \quad . \qquad \text{(xx) (a)}$$

Similarly, if helium be the lifting gas, we find, by a simple change of constants in the above, that:—Gross lift of container filled with helium

$$=[24.971\ gVKP/RT]\{1-(.46\ r_A\pi_T+.54\ r_G\pi_{T(t)})/P\} \quad . \qquad (xx)\ (b)$$

It is convenient at this stage to examine the various terms involved in the above expressions in somewhat more detail, before proceeding to the development of practical lift formulæ. Ignoring the absolute constants, these are:—

(a) *Pressure*

P here signifies the pressure of the atmosphere at the height of the C.B. of the container. As shown in paragraph 3 for that of an airship, the maximum difference between this pressure and that of a barometer in the control car is only about ½ per cent., so that in ordinary circumstances the difference can be neglected. In the case of a "Lift and Trim" trial, however, there is no reason why the barometer should not be placed at a level about half-way up the ship's side, and this is usually done. Since during such a trial all gas containers are "full," this level corresponds very nearly to the height of the C.B. of each of the latter.

In such a case also care must be taken that the readings of the barometer are not "reduced to mean sea-level" as it might be possible for this correction to be unconsciously inserted amongst the others when converting "inches or mm. of mercury" into millibars.

(b) *Temperature*

In a similar way T denotes the absolute temperature of the atmosphere at the level of the C.B. of the container, and for that of an airship this value may differ by a possible ½ per cent. from that observed at the control car. In "Lift-and-Trim" trials the error can be eliminated, of course, by taking the mean of the temperatures at the top and bottom of the shed, or preferably the mean of a series of readings from thermometers situated in various symmetrical positions about the central axis of the airship.

Note that since gross lift is proportional to the atmospheric pressure divided by temperature, the two errors just mentioned tend to cancel out if the "lapse-rate" be positive, since both temperature and pressure then decrease with height. If, however, there be a large "inversion" of temperature gradient, such as has been remarked to exist on occasions inside a shed, then the above does not hold, and the errors are additive; the net result is that the gross lift may be over-estimated by as much as one per cent.

(c) *Purity of Gas*

K denotes the "dry purity" as defined in paragraph 2, and varies very slowly with time. This is because any "valving" of gas releases hydrogen (or helium) and air molecules in very nearly the same proportions as they already exist in the container, the gas being assumed to be almost completely mixed. Any change in K can only therefore occur through differential filtration through the walls, and with good gas fabric or in a metal-clad ship this is very small.

The conventional definition of "purity"[13] is that it is the ratio of the lift of a container filled with the impure and moist gas, to that which it would have under the same conditions of pressure, temperature, and atmosphere humidity if the gas were absolutely pure and dry. We can easily find what the lift would be under the latter conditions by putting $r_G=0$ and $K=1$ in equation (xx) (*a*) or (*b*). In this case we should have

[13] See the paper, already referred to, by Scott and Richmond. Appendix III.

Gross lift of container filled with pure and dry hydrogen

$$= (26.955\ gVP/RT)\ \{\ 1 - .42\ r_A\pi_T/P\ \}$$

Hence " purity " as usually defined $= K'$ (say)

$$= \frac{K\ [(26.955\ gVKP/RT)\ \{\ 1 - (.42\ r_A\pi_T + .58\ \pi_{T+t})/P\ \}\ ]}{[(26.955\ gVP/RT)\ \{\ 1 - .42\ r_A\pi_T/P\ \}\ ]}$$

$$\text{or } K' \text{ for hydrogen} = K\ (1 - .58\ r_G\pi_{T+t}/P)$$

by the Binomial Theorem, ignoring, as before, squares, etc., of $.42\ r_A\pi_T/P$. Similarly

$$K' \text{ for helium} = K\ (.54\ r_G\pi_{T+t}/P)$$

Thus K' is affected by the humidity conditions inside a gas container and since these can vary very rapidly, especially with change of temperature (due to alteration of height or otherwise), it cannot be considered, strictly speaking, a constant to the same extent that K can. Hence the more logical definition is adopted here, K being called the " dry purity," since if the lifting gas be perfectly dry and $r_a = 0$, the two definitions agree or $K = K'$, as can be seen from the above equations.

Up to the present, however, measurements of " purity " have been made in terms of K', so that in such cases to find K it is necessary to make a correction, which from transposition of the above equations is given by

$$K = K'\ (.58\ r_G\pi_{T+t}/P) \text{ for hydrogen} \quad . \quad . \quad \text{(xxi) } (a)$$

$$K = K'\ (.54\ r_G\pi_{T+t}/P) \text{ for helium} \quad . \quad . \quad . \quad \text{(xxi) } (b)$$

The term $r_G\pi_{T+t}/P$ is, strictly speaking, a function of these variables r_G, π_{T+t}, and P, but of these P does not vary very greatly. As we have seen, p can be completely neglected in comparison with P, as " purity " measurements are almost invariably taken at ground level. It will be sufficient, therefore, to make P constant and equal to 1,000 mb. (the change of P from mean sea-level to 5,000ft. is only about 17 per cent. and " seasonal " changes on either side of the mean about ± 8 per cent.). As, moreover, π_{T+t} is a function only of $(T+t)$, or the temperature of the lifting gas, we can plot, for various values of the relative humidity (*e.g.*, for every 20 per cent.), the correction directly against this temperature. The result is shown in Graph D, which serves with sufficient accuracy for both hydrogen and helium, taking the mean numerical factor as .56. Since K' is in general very nearly equal to unity, it is sufficient to add simply the percentage correction to the " percentage purity " as commonly defined in order to find the " percentage dry purity," and the graph has been scaled accordingly.

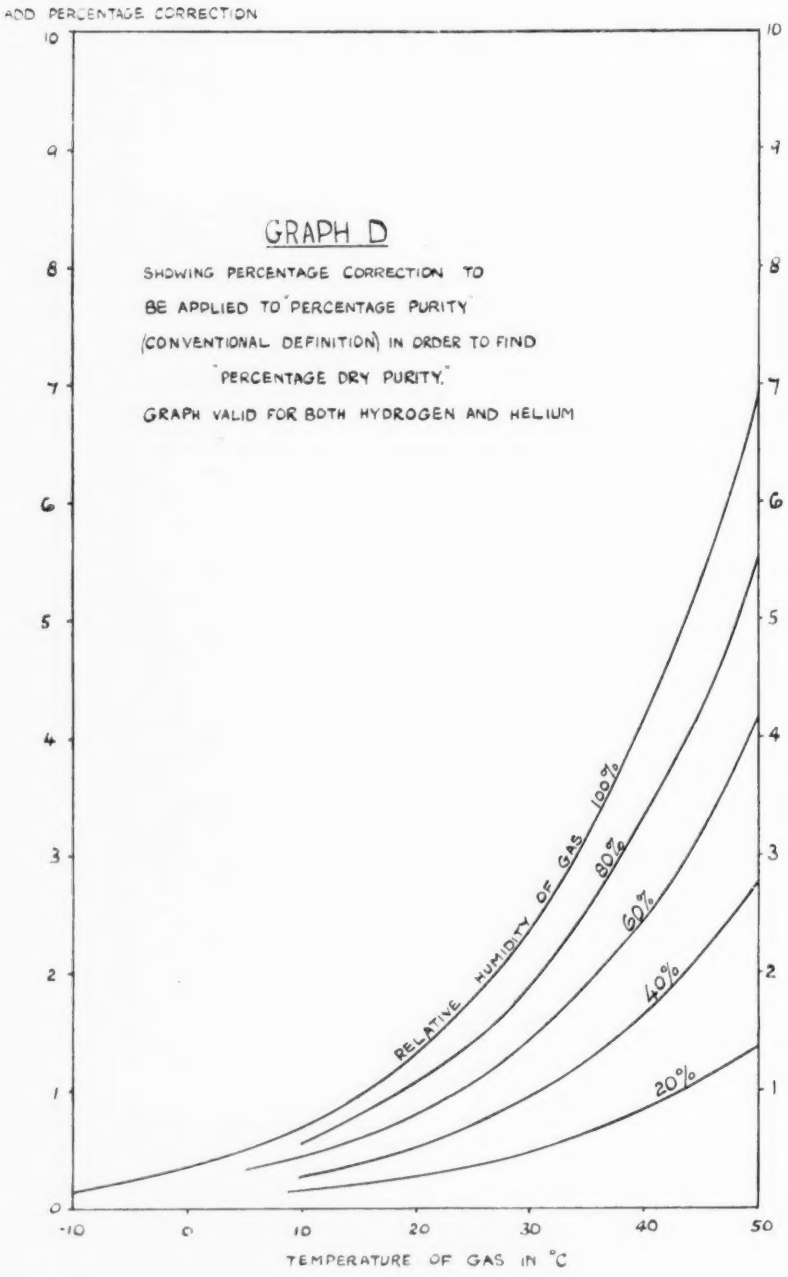
It is hardly necessary to point out that the values of r_G and π_{T+t} used in such corrections are the actual values, of course, at the time of the " purity " measurements.

The graph shows in very striking fashion how it is possible, even in an ideally gas-tight container, for K' to vary very considerably, particularly in hot and humid climates. Thus, suppose that such a container were initially at ground level and its gas were saturated at temperature 40°C. If now the container be taken to an altitude such that the temperature of the gas falls to 20°C., then the latter will be still saturated, but K' will have decreased by about 2 per cent. On the other hand, K would be sensibly the same at both levels, which demonstrates one advantage of using it as a factor instead of K'.

Another advantage is that " purity " meters actually measure K most easily (see Appendix II), so that to find K' additional though minor extra calculation is involved.

(*d*) *Volume*

Since in airships as at present constructed the gas containers are in practically all cases " flexible," and by this is meant that they are to a certain extent free to expand or contract in order to accommodate themselves to changing condi-


ADD PERCENTAGE CORRECTION
GRAPH D
SHOWING PERCENTAGE CORRECTION TO
BE APPLIED TO "PERCENTAGE PURITY"
(CONVENTIONAL DEFINITION) IN ORDER TO FIND
"PERCENTAGE DRY PURITY."
GRAPH VALID FOR BOTH HYDROGEN AND HELIUM
RELATIVE HUMIDITY OF GAS 100%
80%
60%
40%
20%
0
1
2
3
4
5
6
7
8
9
10
-10
0
10
20
30
40
50
TEMPERATURE OF GAS IN °C

tions of " super-pressure," it is, therefore, customary to consider the volume V of such a container at any time in relation to the maximum volume V_0 (say) which it can occupy at extreme extension. The ratio of the two, expressed as a percentage, is called the " percentage fulness," and will be denoted here by 100λ. Hence we have the simple relation

$$V = \lambda V_0 \qquad \text{(xxii)}$$

The above definition is perfectly logical, but another one is sometimes given[14] which is not quite equivalent, although the practical difference is too small to be noticeable (see paragraph 10).

The accurate estimation of λ is a matter of some difficulty and is in general almost impossible, although a " past " value can sometimes be inferred from a knowledge of gas and atmospheric conditions at that time and when the container is " full." Various attempts[15] have been made to deduce the theoretical form which a more or less " free " balloon takes up under various conditions of pressure, etc., but save in simple cases such as that of a solid of revolution, without much success. The problem is naturally complicated by the lack of symmetry about a vertical axis which is shown by an airship, and by the irregularity of shape which a fabric container takes up in general, so that a comprehensive solution on these lines is not likely to be found.

In practice, therefore, λ is usually estimated " by eye," giving results which cannot usually be depended upon within less than 5 per cent. For this reason all reasonably accurate determinations of the gross lift of an airship are made when its gas containers are full, *i.e.*, either at " pressure height," or when lying fully " gassed " in a shed, and normally one of the first operations after commencing a voyage is to " find pressure height " for this purpose.

(e) *Humidity*

The factor $[1-(.42\,r_A\pi_T+.58\,r_G\pi_{T.t})/P]$ in equation (xx) (*a*) represents (since K is by definition independent of humidity) the direct physical effect on the lift of any water-vapour present in the atmosphere and lifting gas (in this case hydrogen). This factor is of similar form to the one just discussed, with the additional complexity that there are, strictly speaking, five variables (*i.e.*, r_A, r_G, π_T, $\pi_{T.t}$ and P) to be considered. Fortunately, however, for practical purposes it can be very much simplified.

In the first place, negligible error is introduced by making P constant and equal to 930 mb., this being the average atmospheric pressure at height 2,250ft. The latter altitude has been chosen for several reasons, the principal one being that in the future large commercial airships may be expected to operate generally in the vertical zone between 0 and 5,000ft., and for lift calculations it is therefore a good average. Moreover, at ground level the chosen value of P is at most only about 10 per cent. too small. Then this height is approximately the upper limit of any " inversion " in the atmospheric " temperature gradient," and is the greatest altitude, therefore, at which one might expect to find temperatures of the same order as those at ground level, and hence large values of $r_A\pi_T$ and $r_G\pi_{T.t}$. On ascending further the " lapse-rate," though possibly irregular, will be positive, and hence vapour pressures, both in the atmosphere and lifting gas, will fall rapidly, in accordance with Graph C; this makes the factor more and more equal to unity, and hence negligible. The above is confirmed by an analysis of Humphreys,[16] who has demonstrated that on an average the vapour pressure in the atmosphere at 6,000ft. is only about one-half of that at ground level.

We can now convert the factor into the form

$$[1-(.42\,r_A\pi_T+.58\,r_G\pi_{T.t})/930 \text{ mb.}],$$

[14] Cf. " The Effect of Meteorological Conditions on Airships," para. 1 (a).
[15] *e.g.*, Crocco's solution given in Blakemore and Pagon's " Pressure Airships."
[16] W. J. Humphreys, " Physics of the Air," p. 72.

which can also be written as

$$[1-\tfrac{1}{2}(r_A\pi_T+r_G\pi_{T+t})/930 \text{ mb.} + .08\,(r_A\pi_T - r_G\pi_{T+t})/930 \text{ mb.}].$$

The latter term is greatest when $r_A\pi_T$ is a maximum and $r_G\pi_{T+t}=0$, or vice-versa and since the maximum of $r_A\pi_T/930$ mb. or $r_G\pi_{T+t}/930$ mb. is about .13, that of the term is about .01, so that at most a one per cent. error is involved in neglecting it altogether.

It has been proved experimentally[17] that water vapour diffuses through gas fabric about 2,500 times as fast as hydrogen, and as in all airships, even if "metal-clad," there is at some place a fabric surface separating the atmosphere from the lifting gas, it might be expected (and has been practically demonstrated) that there is a tendency for a state of equilibrium to be established such that the vapour pressures on either side become equal. This means that the quantity $r_A\pi_T$ tends to become equal to $r_G\pi_{T+t}$. If the ship has been "gassed" some time previously we can without serious error take r_G as the same as r_A.

From both these considerations it is evident that the term

$$.08\,(r_A\pi_T - r_G\pi_{T+t})/930 \text{ mb.}$$

can be neglected altogether, so that the "humidity" factor reduces to

$$[1-\tfrac{1}{2}(r_A\pi_T+r_G\pi_{T+t})/930 \text{ mb.}].$$

In the case of a helium-filled container the corresponding factor

$$1-(.46\,r_A\pi_T+.54\,r_G\pi_{T+t})/P$$

may be considered in exactly the same way, the mean of .46 and .54 being also .5. We thus have:—

"Percentage humidity factor" for hydrogen or helium

$$=100\,\eta \text{ (say)}$$
$$=100-\tfrac{1}{2}(100\,r_A\pi_T+100\,r_G\pi_{T+t})/930 \text{ mb.}$$
$$=100-(\text{mean of } 100\,r_A\pi_T/930 \text{ mb. and } 100\,r_G\pi_{T+t}/930 \text{ mb.}) \qquad \text{(xxiii)}$$

Each of the terms $100\,r_A\pi_T/930$ mb. and $100\,r_G\pi_{T+t}/930$ mb. is of the form (relative humidity × saturated vapour pressure in mb./930 mb.), and can thus be taken from one graph or set of tables by making the appropriate substitutions. Graph E has been plotted accordingly, to show their values for different ones of this relative humidity and temperature (since saturated vapour pressure is a function of temperature only); the scale on the left hand side gives the terms directly and the method of use of the graph is obvious.

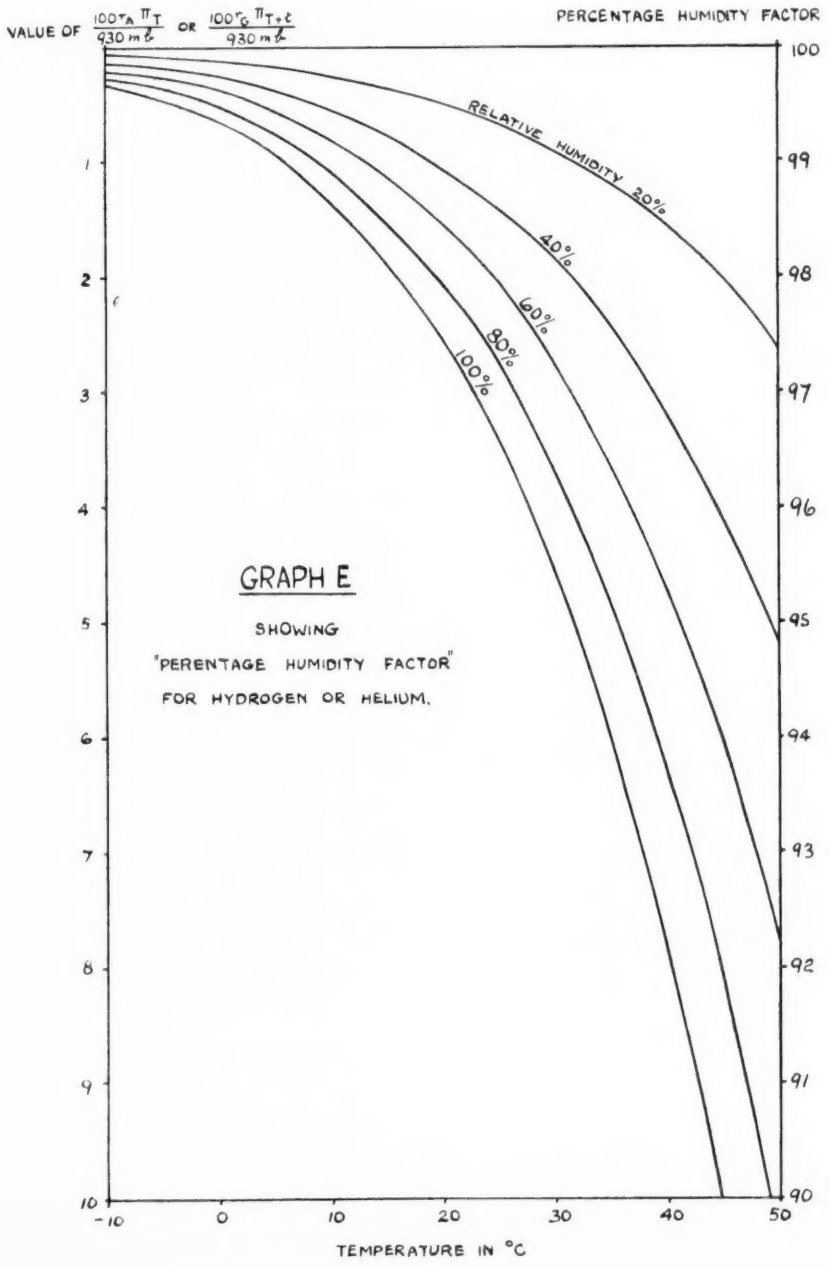
In most cases, however, and unless the difference of air and gas temperatures is greater than about 5°C., it is sufficiently accurate to assume that (mean of $100\,r_A\pi_T/930$ mb. and $100\,r_G\pi_{T+t}/930$ mb.) is the same as (mean relative humidity of gas and air × saturated vapour pressure at mean temperature of latter/930 mb.) in which case the "percentage humidity factor" can be read in one operation from the scale on the right hand side.

It will be noticed that here again the correction, whilst only small in temperate climates, becomes of importance in hot and moist areas, *e.g.*, for saturation of air and gas at 45°C. the "percentage humidity factor" is 90, and there is a 10 per cent. loss of lift due to the water vapour.

The relative humidity of the air can be determined easily by a hygrometer, but in the case of the gas a direct measurement is obviously somewhat more difficult. Nevertheless it is usually possible to make a fairly accurate estimate from a consideration of the conditions at the time.

On the other hand, immediately after "gassing," especially if by the "Silicol" process, the relative humidity of the gas is probably nearly 100 per cent. The latter applies also at "pressure height," where, due to fairly rapid cooling in ascent, it is almost certain that the gas will be saturated or nearly so. Lacking, therefore, any instrumental evidence η can be estimated on these lines.

[17] Vide A.R.C. Report 81.


VALUE OF $\frac{100\tau_A \pi_T}{930 m_b}$ OR $\frac{100\tau_G \pi_{T+t}}{930 m_b}$
PERCENTAGE HUMIDITY FACTOR
RELATIVE HUMIDITY 20%
40%
60%
80%
100%
GRAPH E
SHOWING
"PERENTAGE HUMIDITY FACTOR"
FOR HYDROGEN OR HELIUM.
1
2
3
4
5
6
7
8
9
10
100
99
98
97
96
95
94
93
92
91
90
-10
0
10
20
30
40
50
TEMPERATURE IN °C

We can now return to equations (xx) (*a*) and (*b*) and write them, with appropriate substitutions from equations (xxii) and (xxiii) in the simple form:—

Gross lift of container filled with hydrogen

$$=(26.955\ g/R)\ (P/T)\ K\lambda V_o\eta$$

Gross lift of container filled with helium

$$=(24.971\ g/R)\ (P/T)\ K\lambda V_o\eta$$

Let now P_o and T_o denote the pressure and temperature at mean sea-level in the chosen "standard atmosphere"; for the one which has been here selected (I.C.A.N.) these will be 1013.2 mb. and 15°C. or 288°A. respectively. Then we can write the above equations in the form:—

Gross lift of container filled with hydrogen

$$=[26.955\ gP_oV_o/RT_o]\ [(P/P_o)\ (T_o/T)\ K\lambda\eta]$$

Gross lift of container filled with helium

$$=[24.971\ gP_oV_o/RT_o]\ [(P/P_o)\ (T_o/T)\ K\lambda\eta]$$

By putting $V_o=K=\lambda=\eta=1$, $P=P_o$ and $T=T_o$, it is easily seen that the expression $26.955\ P_o/RT_o$ is the lift in weight units of unit volume of pure and dry hydrogen at mean sea-level in the "standard atmosphere"; similarly for the corresponding expression in the case of helium. Then can be easily calculated numerically, giving:—

Gross lift of container filled with hydrogen

$$=(71.20 \text{ lbs./1,000 cu. ft.})\ \{\ V_o\ (P/1013.2 \text{ mb.})\ (288°\text{A.}/T)\ K\lambda\eta\ \}$$
$$=(31.79 \text{ tons}/10^6 \text{ cu. ft.})\ \{\ V_o\ (P/1013.2 \text{ mb.})\ (288°\text{A.}/T)\ K\lambda\eta\ \} \quad \text{(xxiv) } (a)$$

and gross lift of container filled with helium

$$=(65.96 \text{ lbs./1,000 cu. ft.})\ \{\ V_o\ (P/1013.2 \text{ mb.})\ (288°\text{A.}/T)\ K\lambda\eta\ \}$$
$$=(29.44 \text{ tons}/10^6 \text{ cu. ft.})\ \{\ V_o\ (P/1013.2 \text{ mb.})\ (288°\text{A.}/T)\ K\lambda\eta\ \} \quad \text{(xxiv) } (b)$$

Helium has thus about $7\frac{1}{2}$ per cent. less lift than hydrogen.

Finally, from the above, the following practical formula can be deduced:—

Gross lift of container = (constant) × (gas volume in millions of cubic feet) × (pressure of air in mb.) × (percentage dry purity) × (percentage fulness) × (percentage humidity factor) ÷ (temperature of air in °A.) × 10^6 tons, where the constant

$$=9.035 \text{ for hydrogen } (\log_{10} 9.035=.95592)$$
$$=8.370 \text{ for helium } (\log_{10} 8.370=.92272) \quad . \quad . \quad \text{(xxv)}$$

This is an accurate and convenient relation well adapted to the use of logarithms or slide rule.

In the case of an airship which, like most rigids of the conventional type, has a series of independent gasbags, the total gross lift is obviously the sum of the lifts of each, and can be thus found, knowing the individual gas volumes. The pressure and temperature of the air, also (practically) the "percentage humidity factor," are the same for each and, once found, can be used as a constant multiplier. Thus:—

$$\text{Gross lift of airship} = \frac{\text{Constant} \times (\text{pressure of air in mbs.}) \times \text{percentage humidity factor}}{(\text{Temp. of air in °A.}) \times 10^6 \times \Sigma\ (\text{gas vol. in millions of c.f.}) \times (\text{percentage fulness})}$$

where the Σ denotes summation for all the gasbags, and the constant is as in equation (xxv).

As already noticed, however, owing to uncertainty in knowledge of the value of "percentage fulness," the gross lift of any gasbag can only be determined accurately when full and just "blowing-off" gas, *i.e.*, (*a*) at a "Lift and Trim" trial, (*b*) at "pressure height" for the bag.

In case (*a*) all bags are full simultaneously so that one can define a " mean percentage dry purity " given by $100\,K = (100\,K_1V_1 + 100\,K_2V_2 + \ldots)/V_0$ where $100\,K_1 \ldots$, $V_1 \ldots$, are the individual " percentage dry purities " and the individual " full " gas volumes and $V_0 = V_1 + V_2 + \ldots$ is the maximum total gas volume of the ship.

Equation (xxvi) now becomes, therefore, putting " percentage fulness " in each case equal to 100,

> Gross lift of an airship with full containers = constant × (total gas volume in millions of cubic feet) × (pressure of air in mb.) × (mean percentage dry purity) × (percentage humidity factor) /(temperature of air in °A.) × 10^4 tons . . (xxvii)

where the constant is as in equation (xxv).

In case (*b*) each bag will in general " blow-off " at a slightly different height, owing to initial difference in their " percentage fulness " and the pressure adjustment of the automatic valves. In order to avoid undue loss of gas efforts are usually made to keep this " zone of pressure heights " as narrow as possible for the sake of stability, the adjustments being such that, if any, the end bags " blow-off " first.

Strictly speaking, then, the lift of each gasbag should be determined separately, using the formula given by equation (xxv), since the pressures and temperatures of the atmosphere corresponding to each " pressure height " differ slightly. As this would be excessively tedious in practice, however, the number of bags usually exceeding 12, it is customary to estimate a " mean pressure height," whence the corresponding atmospheric pressure can, if necessary, be found from Graph A if the altimeter from which the readings are taken be " uncorrected I.C.A.N." Similarly the appropriate air temperature can be deduced from the mean readings of an air thermometer during the operation, and lastly the purity factor to be used is equivalent to " mean percentage dry purity," as already defined. Hence, equation (xxvii) can be used for this case.

If all the gasbags were of equal volume, the errors involved in this procedure would tend mutually to cancel out, but as they are not, there is bound to be a certain discrepancy. This cannot be large, however, since it is easy to show from equations (xvii) and (xviii) of paragraph 5 that at moderate altitudes the pressure and temperature in an average atmosphere represented by the I.C.A.N. Convention only decrease at a rate of about .36 per cent. and .07 per cent. respectively per 100ft.

Equation (xxvii) is thus accurate under " Lift and Trim " trial conditions and very nearly so when estimating gross lift by observations of " pressure height."

Lastly, it may be necessary on some occasions, such as at the commencement of a flight, to find the gross lift when the gas containers are only partially full, and when no previous records are available. In this case the " percentage fulness " of each of the former is estimated " by eye " and a " mean percentage fulness " deduced given by $100\,\lambda = (100\,\lambda_1V_1 + 100\,\lambda_2V_2 + \ldots)/V_0$ where $100\,\lambda_1$, $100\,\lambda_2 \ldots$, etc., are these individual values and V_0, $V_1 \ldots$, etc., are as above; in practice the calculation can be rapidly effected provided that there is a table handy giving the percentage that the volume of each container when full bears to the total gas volume of the ship.

In view of the uncertainty in the value of this " mean percentage fulness " it is useless in this case to retain the " percentage humidity factor," so that we have:—

> Gross lift of airship very approximately = constant × (total gas volume in millions of cu. ft.) × (pressure of air in mb.) × (mean percentage dry purity) × (mean percentage fulness)/(temperature of air in °A.) × 10^4 tons . . . . (xxviii)

where the constant approximately equals 9 for hydrogen and 8.4 for helium.

7. *Reduction to Standard Conditions*

For convenience in design and for use as a basis of comparison, it is customary to specify a " standard " gross lift for an airship, this being the lift which the latter would have at mean sea-level in some chosen " standard atmosphere," with (*a*) all gas containers full and just " blowing-off," (*b*) the atmosphere and lifting gas both dry and at the same temperature, (*c*) the gas at a certain specified purity.

In Great Britain, this purity specified for hydrogen-filled airships is that which gives a lift of 68lbs. per 1,000 cu. ft. in a dry atmosphere at 9°C. and 1,014 mb., from which it is easily deduced (equation (xxiv) (*a*)) that :—

" Standard percentage dry purity " for hydrogen-filled ships = 93.5% (xxiv) (*a*)

This is a value somewhat lower than usually found in practice, and is therefore on the " lenient " side. Such is quite justified, however, since it is almost invariably found that the weight of an airship when completed is greater than allowed for in design.

As regards helium, the corresponding purity specified in the United States appears to be that which gives a lift of 62lbs. per 1,000 cu. ft. in a dry atmosphere at 60°F. and 29.92in. of water,[18] from which it is also deduced (equation (xxiv) (*b*)) that :—

" Standard percentage dry purity " for helium-filled ships = 94% (xxiv) (*b*)

In marine design it is comparatively easy to determine with accuracy the weight of a water-borne vessel given the " curve of displacement " (deduced from the geometrical form of her hull) and the density of the water. On the other hand, estimates based upon knowledge of the weights of the various components put into the structure are liable to be seriously in error. In the case of an airship, however, it is probable that the " integrated fixed weights " can be evaluated with much greater accuracy, but, on the other hand, the determination of total gas volume is liable to be in error, due to uncertainty of the form taken up by the flexible containers. In general, therefore, there will be a discrepancy revealed at a " Lift and Trim " trial between the lift deduced from knowledge of the total gas volume and the physical conditions at the time, and that determined by summing up the weights on board, for which discrepancy it is difficult à priori to assign the cause, as the errors of the above are about of the same order.

Assuming, however, that the estimated total gas volume is correct, it is quite easy to deduce the gross lift in " standard " conditions from the equations already established. For taking equation (xxiv) (*a*) for the case of a hydrogen-filled ship, and remembering that under the conditions above specified $\lambda = 1$ for all the containers, K, the " mean dry purity " $= K_0$, the " standard mean dry purity," and $\eta = 1$, we find that gross lift of hydrogen-filled airship under " standard " conditions :—

$$= (31.79 \text{ tons}/10^6 \text{ cu. ft.}) \{ V_0 (P/1013.2 \text{ mb.}) (288°\text{A.}/T) K_0 \}$$

where V_0 is now the total gas volume and P and T refer to the particular " standard atmosphere " chosen. For the " I.C.A.N. " case with $K_0 = .935$ this further reduces to :—

Gross lift of hydrogen-filled airship under " standard " I.C.A.N. conditions
$= [29.72 \times (\text{total gas volume in millions of cu. ft.})]$ tons
$(\log_{10} 29.72 = 1.47304)$ . . . (xxix) (*a*)

Similarly, in the case of helium :—

Gross lift of helium-filled airship under " standard " I.C.A.N. conditions
$= [27.66 \times (\text{total gas volume in millions of cu. ft.})]$ tons
$(\log_{10} 27.66 = 1.44185)$ . . . (xxix) (*b*)

[18] Vide Paper on " The Metalclad Airship " read by C. B. Fritsche before the R.Ae.S. on May 14th, 1931.

If the alternative assumption be made that the total integrated weight W at the time of the trial is correct, then any discrepancy between this and the calculated " actual " gross lift L can only be attributed to error in the estimation of total gas volume V_0, since the other quantities involved in the " lift equation " are known comparatively accurately. Hence we can now determine a better approximation to the value of the volume given by $W/L \times V_0$, from which a corrected value of the " standard " gross lift can be computed. In practice, since $(L-W)$ is small, this simply involves subtracting the latter from the value originally determined.

It will be noticed that, contrary to a fairly widespread belief, superheating does not affect either the " actual " or " standard " gross lifts of an airship when her gas containers are full. This may seem somewhat surprising, but can be justified from à priori reasoning as follows:—

As under the latter conditions the gas containers are about to " blow-off, " they can be replaced in imagination by rigid ones of constant volume. Suppose now we consider an airship so constructed and situated at mean sea-level in an I.C.A.N. " standard atmosphere." When filled with helium it has, as already shown, about $7\frac{1}{2}$ per cent. less lift than hydrogen under the same conditions. But helium is almost exactly twice as dense as hydrogen, so that to make the lift the same in the two cases we must, for a container of constant volume, artificially halve the helium density. From the " density equation " this can only be effected by doubling its absolute temperature, which is equivalent to superheating it 288°C.

Hence, superheating the gas 288°C. only increases the lift about $7\frac{1}{2}$ per cent., so that it is easily seen that for the values occurring in practice, which are certainly not greater than 25°C., the increase is completely negligible. Incidentally this proves the fallacy of the suggestion sometimes made that the lift of an airship could be increased appreciably by artificially heating its gas. Evidently the result would not be at all commensurable with the weight involved in the necessary apparatus.

The above does not hold in the case of gas containers only partially full, for under the latter circumstances the effect of superheating is of paramount importance, as is well-known. This phenomenon will be further discussed in paragraph 9.

8. *Example of Use of Lift Formulæ*

It will be instructive at this stage to work out a numerical example taken from actual practice, in order to illustrate the method of use and advantages of the formulæ established in the last two paragraphs. This example consists of an analysis of two " Lift and Trim " trials on the airship R.100, the first at Howden on November 11th, 1929, and the second at Cardington on April 22nd, 1930, an interval of just over five months elapsing between them. From the date given for each occasion we find:—

1st Trial

Total gas volume of airship $= 5.155 \times 10^6$ cu. ft.
Pressure of atmosphere 29.2″ of mercury at 0°C = 988.8 mb.
Temperature of atmosphere (dry-bulb) = 47°F. = 8.3°C. = 281.3°A.
Temperature of atmosphere (wet-bulb) = 46.1°F.
Mean temperature of gas = 41.3°F. = 5.2°C.
Mean percentage purity (conventional definition) = 96.7 per cent.

From Glaisher's " Hygrometrical Tables," we find, corresponding to the wet and dry-bulb temperatures, that the relative humidity of the atmosphere = 94 per cent. Since " gassing " by the " Silicol " process had taken place quite recently in order to fill all the gasbags up to " blowing-off " point, and

since, moreover, the former were observed to be dripping wet, it may be assumed that the hydrogen was saturated. Hence the mean relative humidity of the atmosphere and gas was $(100+94)/2$, or 97 per cent. The mean temperature of the gas and air was $(8.3+5.2)/2$, or 6.7 per cent., and corresponding to this we find from Graph E that the " mean percentage humidity factor " is 99 per cent., very nearly.

Again, since gas purity measurements had been made just previously, it may be assumed that the gas was also saturated when these took place, and also that it was then at very nearly the same temperature as that given for the trial. Hence, from Graph D, the correction to the conventional " purity " (in terms of which the measurements were made) to give " mean percentage dry purity " is .5, so that the latter equals 96.7 + .5 or 97.2 per cent.

Hence, from equation (xxvii) of paragraph 6, gross lift of airship

$$=(9.035\times 5.155\times 988.8\times 97.2\times 99)/281.3\times 10^4 \text{ tons.}$$

which on applying five-figure logarithms works out at

157.70 tons.

The total ballast and movable weights on the ship on the day in question were given as 53.93 tons. The fixed weights were therefore apparently 157.70 − 53.93 or 103.77 tons. On the other hand, the total " integrated fixed weights," including an extra fuel system recently installed, were given as 98.33 tons.

Hence there remains a discrepancy of 103.77 − 98.33 or 5.44 tons unaccounted for, due, as hitherto stated, to over-estimation of gas volume and under-estimation of the fixed weights in some unknown proportion. In this case, the former is hardly likely to be appreciably the case, since 5.44 tons corresponds roughly to the lift under " standard " conditions of a volume of 183,000 cu. ft. and it is inconceivable that an over-estimate to this extent could have been made. The deficiency is probably due in some measure to certain weights put on to the structure and only partially allowed for, *e.g.*, paint and varnish, but as shown in paragraph 11, there is good reason for belief that the bulk is due to soakage in the gasbags but not in the outer cover.

If now it is desired to find the gross lift under " standard " I.C.A.N. conditions, this is, assuming $V_0 = 5.155\times 10^6$ cu. ft., equal to 29.72×5.155 or 153.20 tons, from equation (xxix) (*a*).

For the reasons given, this may be accepted as being very near the true value.

2nd Trial

Total gas volume (as before) $= 5.155\times 10^6$ cu. ft.
Pressure of atmosphere = 1003 mb.
Temperature of atmosphere (dry-bulb) = 55.2°F. = 12.9°C. = 285.9°A.
Temperature of atmosphere (wet-bulb) = 46.8°F.
Mean temperature of gas = 57°F. = 13.8°C.
Mean percentage purity (conventional definition) = 95.14 per cent.

As before, from Glaisher's Tables it is found that corresponding to the wet and dry-bulb temperature the relative humidity of the atmosphere is 55 per cent., and making the same assumption as before, that of the gas may be again taken as 100 per cent. Hence the mean relative humidity is $(100+55)/2$ or, say, 77 per cent., and mean temperature is $(12.9+13.8)/2$ or 13.3°C.; corresponding to this we find from Graph E that the " mean percentage humidity factor " is 98.7 per cent. Also from Graph D the correction to the " purity " given in order to find " mean percentage dry purity " is + .92 so that the latter = 95.14 + .92 or 96.06 per cent.

Hence gross lift of airship

$$= (9.035 \times 5.155 \times 1003 \times 96.06 \times 98.7)/(285.9 \times 10^4) \text{ tons}$$
$$= 154.92 \text{ tons.}$$

The total ballast and movable weights at the trial were given as 49.94 tons, so that the total fixed weights were 154.92 − 49.94, or 104.98 tons. Comparing this with the result of the previous trial we see that the ship has apparently gained weight to an extent equal to 104.98 − 103.77, or 1.21 tons. The cause of this is again very obscure, but the magnitude is such that it may be explained as before as being partly due to the putting on the structure of extra materials such as paint, varnish, and dope, partly to extra " soakage " of the fabric during the winter months, and possibly also to a slight readjustment of the gasbags, since the latter do not in practice " bed down " when full in the same position on every occasion.

It is suggested that the above practical example demonstrates the value and simplicity of operation of the various lift formulæ developed in the previous paragraphs, and especially the facility with which they are adopted to logarithmic work.

9. *Variation of Lift in Flight*

(a) Gas Containers Partially Full. The Effect of Superheating.

In connection with the practical navigation of an airship, it is of considerable importance that a pilot should be able to anticipate any variations of gross lift which might occur through change of meteorological conditions, and this if possible without the necessity of making intricate calculations and by the aid of either prepared graphs or tables and a slide-rule. It would be of advantage also if an instrument could be devised which would indicate a record automatically the gross lift at any moment, or at least the variations thereof from a predetermined value.

At the present time, an airship is usually designed to fly normally below " pressure height," with all its gas containers only partially full, thus avoiding loss of gas—an important consideration in a helium-filled ship especially. This case is then the most important to be considered, and can be analysed as follows:

From the fundamental equation (xvi) we have:—

$$\text{Gross lift of container} = gV(\rho_A - \rho_G).$$

Now ρ_G may be split up into three " partial " densities, as in the method of deriving equations (xi) and (xii). Denote by ρ_{GH} the " partial " density of the hydrogen (or helium) alone in the container; then the above equation can be written in the form:—

$$\text{Gross lift of container} = [gV\rho_{GH}]\,[(\rho_A - \rho_G)/\rho_{GH}].$$

But from the expression immediately preceding equation (xx) (*a*), we have, if the lifting gas be hydrogen, and using the notation of paragraph 6:—

$$(\rho_A - \rho_G) = [26.955\,K\,(P+p)/R\,(T+t)]\,[1 + (28.971/26.955\,K)\,(t/T) - (28.971/26.955\,K)\,(p/P) - (10.955/26.955\,K)\,(r_A\pi_T/P) - (r_G\pi_{T+t}/P) + (10.955/26.955\,K)\,(r_G\pi_{T+t}/P)]$$

$$\text{Also } \rho_{GH} = [2.016\,K\,(P+p)/R\,(T+t)]\,[1 - (r_G\pi_{T+t}/P)]$$

as in the derivation of equation (xi)

$$\text{Hence } [(\rho_A - \rho_G)/\rho_{GH}] = [26.955/2.016]\,[1 + (28.971/26.955\,K)\,(t/T) - (28.971/26.955\,K)\,(p/P) - (10.955/26.955\,K)\,(r_A\pi_T/P) - (r_G\pi_{T+t}/P) + (10.955/26.955\,K)\,(r_G\pi_{T+t}/P)]/[1 - r_G\pi_{T+t}/P)]$$

The above expression can be simplified by making the same or similar assumptions as those already justified in paragraph 6. These are:—(*a*) Since $(28.971/26.955\,K)$ multiplies the small quantity (t/T), K may be considered constant and equal to .96, so that the expression $(28.971/26.955\,K)\,(t/T)$ can be put

equal to $1.12\ (t/T)$; (*b*) since the container is substantially free to expand in this case, p is extremely small and the expression $(28.971/26.955\ K)\ (p/P)$ can be neglected altogether; (*c*) for similar reasons to those just advanced in (*a*) $(10.955/26.955\ K)$ can be made constant and equal to .42; (*d*) all the terms except unity within the square brackets are so small that their squares, etc., may be neglected, and, applying the Binomial Theorem, we thus find that:—

$$\text{Gross lift of hydrogen-filled container} = [(26.955/2.016)\ (gV\rho_{GH})] \times [1 + 1.12\ (t/T)] \times [1 + \{ .42\ (r_G\pi_{T+t} - r_A\pi_T)/P \}] \quad . \quad \text{(xxx) (a)}$$

But the expression $(gV\rho_{GH})$ represents the weight of hydrogen in the container and is therefore sensibly constant under the assumed conditions, neglecting loss by diffusion and supposing there to be no "valving." If, then, $t=0$ and $r_G\pi_{T+t} = r_A\pi_T$, *i.e.*, if there be no superheating or super-cooling of the lifting gas and the vapour-pressure of water in the latter and in the atmosphere be equal, the gross lift of a container below its "pressure height" is constant under all conditions of temperature, altitude, etc. The expression $[(26.955/2.016)\ (gV\rho_{GH})]$ thus represents the "true" lift as commonly defined, the factor $[1 + 1.12\ (t/T)]$ the effect of superheating or super-cooling, and $[1 + \{ .42\ (r_G\pi_{T+t} - r_A\pi_T)/P \}]$ that of what may be called "differential humidity."

Similarly, by a change of constants, we find that:—

$$\text{Gross lift of helium-filled container} = [(24.971/4)\ (gV\rho_{GH})] \times [1 + 1.16\ (t/T)] \times [1 + \{ .46\ (r_G\pi_{T+t} - r_A\pi_T)/P \}] \quad . \quad \text{(xxx) (b)}$$

If we now substitute for ρ_{GH}, the "true" lift of a hydrogen-filled container is given by $[26.955\ gVK\ (P+p)/R\ (T+t)] \times [1 - r_G\pi_{T+t}/P)]$ and of a helium-filled one by $[24.971\ gVK\ (P+p)/R\ (T+t)] \times [1 - r_G\pi_{T+t}/P)]$. These are analogous expressions to those given in equations (xx) (*a*) and (*b*), and by similar methods, therefore, to the ones adopted for converting the latter into the "practical" formula of equation (xxv) we can prove that:—

"True" lift of gas container = (constant) × (maximum gas volume in millions of cubic feet) × (pressure of air in mb.) × (percentage dry purity) × (percentage fulness) × (percentage *gas* humidity factor)/(temperature of *gas* in °A) × 10^6 tons, where the constant is as in equation (xxv) . . . . . (xxxi)

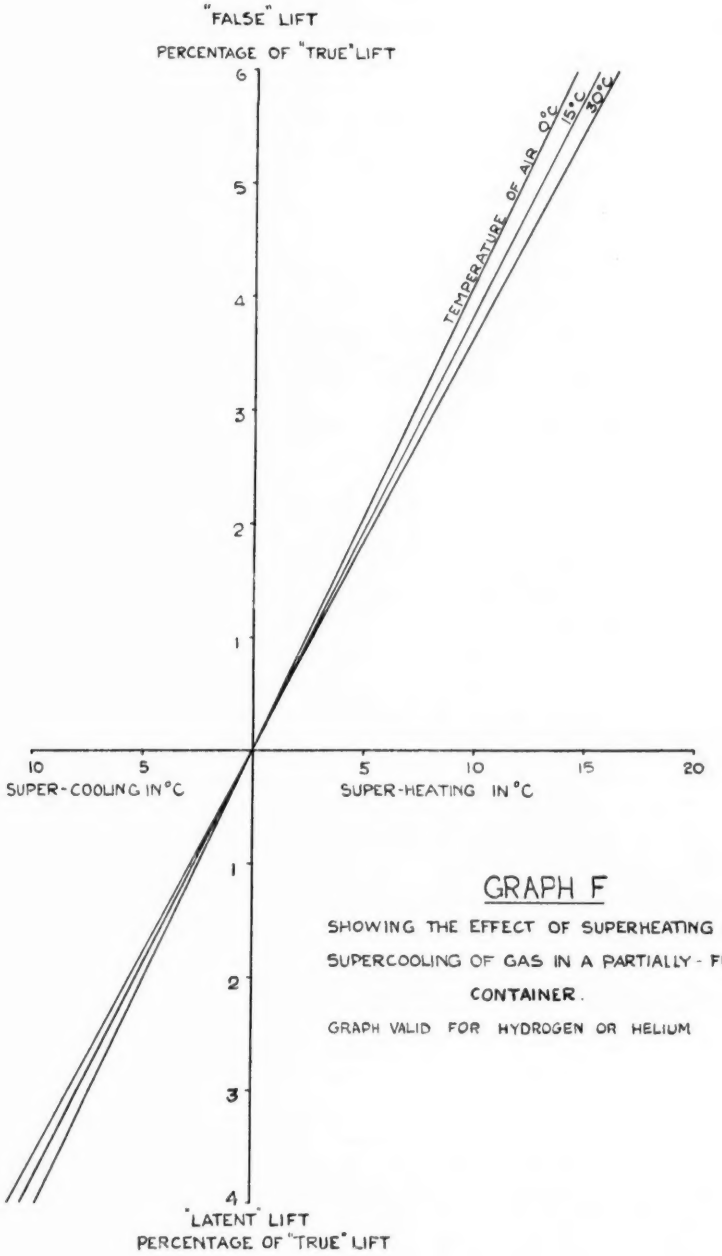
The "percentage gas humidity factor" is defined as the expression $[100 - (100\ r_G\pi_{T+t}/930\ \text{mb.})]$, and can be read in an obvious manner from Graph E, whilst the effect of super-pressure in the gas is neglected as above.

This formula is, however, more or less of academic interest only, as in practice the "true" lift of an airship can be more conveniently deduced from the calculated gross lift by applying a correction.

The expression $[112\ (t/T)] \times [\text{"true" lift}]/100$ evidently represents the "false" lift (or "latent" lift if t be negative in sign) of a hydrogen-filled gas container due to superheating or super-cooling, the corresponding expression for a helium-filled one being $[121\ (t/T)] \times [\text{"true" lift}]/100$. The terms $[112\ (t/T)]$ and $[121\ (t/T)]$ are, however, so nearly the same that clearly no useful purpose could be served by preparing a separate series of tabulated values for the two gases; and they can thus be "meaned" as $[116\ (t/T)]$. Graph F shows the latter plotted for intervals of $T = 15°\text{C.}$, and it will be noticed that changes in the value of T have a comparatively small effect, so that interpolation is easy.

From this Graph it is obvious that quite normal superheating has a considerable effect on the gross lift, as, of course, is well-known in practical experience. For instance, if the temperature of the air be 15°C., and that of the gas 20°C. (a superheating of 5°C.), then the increase on "true" lift is 2 per cent.

The difference of air and gas temperatures should be measured, strictly speaking, at the height of the C.B. of the container, as required by theory. In an airship this may be taken for practical purposes as along the central axis,



GRAPH F

SHOWING THE EFFECT OF SUPERHEATING OR SUPERCOOLING OF GAS IN A PARTIALLY-FULL CONTAINER.

GRAPH VALID FOR HYDROGEN OR HELIUM

since the gas containers are usually kept nearly full in order, of course, to get the maximum possible lift. If, however, this difference be measured at any other position, say the top or bottom of the container, a serious error may be introduced if the " lapse-rates " of the air and gas be of opposite " sign," a phenomenon which it has been shown might occur, for example, in the tropics at mid-day. Fortunately, with the customary " electrical resistance " gas thermometers, a location at or near the central axis presents little difficulty.

Turning next to the " differential humidity " factor, it is evident that the expression $[.42\ (100\ r_G\pi_{T+t} - r_A\pi_T)/P] \times [$" true " lift$]/100$ represents the " false " lift (or " latent " lift if $r_A\pi_T$ be greater than $r_G\pi_{T+t}$) of a hydrogen-filled container due to this effect, the corresponding expression for a helium-filled one being $[.46\ (100\ r_G\pi_{T+t} - 100\ r_A\pi_T)/P] \times [$" true " lift$]/100$. Here again the first terms in each case are very nearly identical, and as in practice the " differential humidity " effect is generally small in comparison with that of superheating or super-cooling, it will be sufficiently accurate to them as both equal to $[\frac{1}{2}\{(100\ r_G\pi_{T+t}/930\text{ mb.}) - (100\ r_A\pi_T/930\text{ mb.})\}]$, in which case their value can be determined very readily from inspection of Graph E.

Combining, then, the two corrections, we have finally :—" False " lift (or " latent " lift if negative in sign) of gas container approximately

$$= [116\ (t/T) + \tfrac{1}{2}\{(100\ r_G\pi_{T+t}/930\text{ mb.}) - (100\ r_A\pi_T/930\text{ mb.})\}] \times [\text{" true " lift}]/100 \qquad \text{(xxxii)}$$

If the walls of a container and envelope of an airship were perfect reflectors of thermal radiation but also (together with the lifting-gas) perfect conductors of heat, then the temperature of the gas would " follow " that of the air without " lag," and there would then be no superheating or supercooling, in addition to which any volume expansions or contractions of the lifting-gas would be under " isothermal " conditions. Similarly, if the walls of the container were perfectly porous to water-vapour, the absolute vapour-pressure of the gas would " follow " that of the air, and there would be no " differential humidity " effect.

In practice, however, the envelope is bound to absorb some of the solar or other radiation which falls upon it, even though such absorption is reduced as much as possible by the application of aluminium paint or other strongly-reflecting substances. The superheating caused in this way is of major importance, and in small airships is the predominating influence which generates " false " lift. Nevertheless, the effect obviously depends, other things being equal, upon the ratio of surface to volume of a container, and therefore not only does it tend to diminish with increase of size of the latter, but also takes a longer time to develop, or there is greater " lag." With the bigger ships of the future, then, we may expect less instability due to this cause—an important argument, incidentally, in favour of building such.

Again, in practice, all gases and fabrics are poor conductors of heat, so that when there occur any rapid variations in the atmospheric pressure, temperature, or humidity (*e.g.*, when changing altitude or traversing a cloud), the transmission of heat to and from the lifting gas is nearly negligible, and the corresponding changes of volume and temperature of the latter are under practically " adiabatic," instead of " isothermal " conditions, the degree of approximation to the former being more and more exact the larger the container, from similar considerations to the above.

Lastly, the diffusion of water-vapour through gas fabric, although much more rapid than that of hydrogen or helium (paragraph 6), is nevertheless still quite slow, for, to quote from the paper of Scott and Richmond, " In the case of a 2,000,000 cu. ft. ship filled with a gas 50 per cent. saturated flying into air which is 100 per cent. saturated, it would take about 35 hours for equilibrium to be established. Such a time lag would obviously be greater in an airship of larger volume, or in a metal-clad ship, so that evidently diffusion can be neglected during the comparatively short period that it takes for an airship to pass, for

example, through a cloud. It follows that sudden variations of vapour pressure inside a container only happen, practically speaking, when there occur the "adiabatic" or nearly "adiabatic" expansions or contractions already mentioned.

Taking all the above factors into consideration then, it is apparent that such expansion or contraction becomes, in increasing measure as larger airships are built, one of the predominating influences governing alterations of gross lift, especially with change of altitude, and hence some analysis of the effect is clearly desirable.

It is shown in any text-book on Thermodynamics, that the equations relating to the "adiabatic" state of an unsaturated gas are, with similar notation to equation (i):—

$$pv^{\gamma} = \text{constant} \quad . \quad . \quad . \quad . \quad . \quad \text{(xxxiii) } (a)$$
$$T = (\text{constant}) \times p^{(\gamma-1)/\gamma} \quad . \quad . \quad . \quad . \quad \text{(xxxiii) } (b)$$

where γ is the ratio of the specific heat of the gas at constant pressure to that at constant volume. For those gases which are of importance in the present connection we have, from Kaye and Laby's "Physical and Chemical Constants," p. 61:

Gas.	γ.	$(\gamma-1)/\gamma$.
Hydrogen	1.419	.296
Helium	1.63	.386
Air	1.401	.286
Water-vapour (at 100°C.) ...	1.368	.270

Consider now a container which is partially full with an unsaturated gas and which is in motion through the atmosphere. We will assume that all changes of state of the former are "adiabatic," also that its absolute temperature and pressure are uniform throughout the volume and the same as at the level of the C.B., and finally that the super-pressure can be neglected. Then with the same notation as at the beginning of this sub-paragraph, and by appropriate substitution in equation (xxxiii) (*b*) we have

$$(T+t) = (\text{constant}) \times P^{(\gamma-1)/\gamma}$$

whence, by taking logarithms and differentiating,

$$\{ 1/(T+t) \} \cdot \{ d(T+t) \} = \{ (\gamma-1)/\gamma \} \ \{ 1/P \} \ \{ dP \}$$
$$\text{or } d(T+t) = \{ (\gamma-1)/\gamma \} \ \{ 1+t/T \} \ \{ T/P \} \, dP$$

For horizontal movements of the container $dP = 0$ sensibly, so that $dt = -dT$, or the increase of virtual superheating is equal numerically to the decrease of atmospheric temperature, and vice-versa. This, of course, is also obvious à priori.

For a small vertical movement dh, $dP = -\rho_A \cdot g \cdot dh = -(PM/RT)\, g \cdot dh$ from equation (vi), where M refers to the atmosphere. Substituting in the above:—

$$d(T+t) = -\{ (\gamma-1)/\gamma \} \ \{ 1+t/T \} \ \{ T/P \} \ \{ PM/RT \} \cdot g \cdot dh$$
$$\text{or } d(T+t)/dh = -\{ (\gamma-1)/\gamma \} \ \{ 1+t/T \} \ \{ gM/R \} \quad . \quad \text{(xxxiv)}$$

From an inspection of this equation it is apparent that the degree of cooling of the lifting gas with ascent of the container and of heating with descent becomes greater, other things being equal, (*a*) the greater the value of $(\gamma-1)/\gamma$, and hence of γ; (*b*) the greater the existing superheating t; (*c*) the greater the value of M. But the latter refers to the atmosphere, and only varies very slightly according to the amount of water-vapour present therein (paragraph 2); from equation (x) its virtual value is seen to be a maximum when the air is perfectly dry.

From the tabulated values of γ and $(\gamma-1)/\gamma$ given above it will be noticed that those for hydrogen and especially helium are larger numerically than the corresponding ones for air and unsaturated water-vapour. It follows from à priori reasoning[19] that dilution of a lifting gas with either or both of the latter reduces

[19] For a more rigid proof see W. J. Humphreys' "Physics of the Air," Chaps. II. and XIII.

the rate at which its temperature falls with " adiabatic " expansion, and vice-versa. If, however, the lifting gas be saturated with water-vapour, the conditions are somewhat more complicated, since any condensation of moisture involves release of latent heat; nevertheless this effect still tends obviously to reduce the cooling or heating (as the case may be) of the whole mass of gas. The equations governing the " adiabatic " and " pseudo-adiabatic " changes of state of saturated air have been given by Hertz[20] and Neuhoff,[21] and similar equations apply to saturated hydrogen or helium. In the main the conclusions to be drawn from their analysis are that for the latter cases relations of the form given by equations (xxxiii) (*a*) and (*b*) do not, strictly speaking, apply, but that the latter are, for the present purpose, a good approximation provided that a lower value of γ be assumed than for the dry gases. Finally, since the actual change of state of the gas in the container is not strictly " adiabatic," due to the small but finite transmission of heat through the walls of the latter, the actual amount of cooling with ascent (and conversely heating with descent) will be somewhat less than that given by equation (xxxiii) (*b*), which is equivalent to assuming a smaller value of $(\gamma-1)/\gamma$, and hence of γ.

The general inference is that the change of temperature of a lifting gas, due to its expansion or contraction, is greatest when it is pure and dry and the existing superheating a maximum, also when the surrounding atmosphere is dry, and the changes are " adiabatic." This being the case, we can easily calculate from equation (xxxiv) the maximum cooling or heating to be expected from this cause.

Assuming that the largest possible value of t/T is .1 (paragraph 6), we have, for pure and dry hydrogen,

$$\{(\gamma-1)/\gamma\}\ \{1+t/T\}\ \{gM/R\} = .296 \times 1.1 \times 981 \text{ cm./sec.}^2 \times (28.971 \text{ gr.} \times {}^\circ\text{A.})/(83.15 \times 10^6 \text{ ergs}) = 11.1{}^\circ\text{C. per km.},$$

or 3.4°C. per 1,000ft., and for pure and dry helium similarly 14.5°C. per km., or 4.4°C. per 1,000ft. It is of interest to note also that if the container were filled with dry air and the existing superheating were zero ($t=0$), then the above expression becomes 9.8°C. per km. or 3°C. per 1,000ft., which is, of course, the " dry adiabatic lapse rate " familiar to meteorologists.

Let us now consider the effect of such " adiabatic " changes on the value of the " gas humidity " term $\frac{1}{2}(100\, r_G\pi_{T+t}/930$ mb.) of equation (xxxii), which term is, more exactly, $.42\ \{100\, r_G\pi_{T+t}/(P+p)\}$ and $.46\ \{100\, r_G\pi_{T+t}/(P+p)\}$ for hydrogen and helium respectively. If either of these gases in a container be unsaturated, then the ratio of $r_G\pi_{T+t}$ to $(P+p)$ is constant during rapid changes of state, since it is simply that of the number of water molecules in vapour form to the total number enclosed, which is invariable when there is no condensation or " valving " and diffusion can be neglected. If, however, the enclosed gas be saturated, then $r_G\pi_{T+t}$ is simply a function of the temperature of the latter, and any such change will now involve also condensation of moisture on the walls of the container, or evaporation from any already present there, so that in addition the gross lift is indirectly affected through the increase or decrease of parasitic weight. The degree to which any deposited moisture is retained depends fundamentally upon the construction of the container, whether of metal or fabric. If the former, such deposits will be comparatively small and will be noticeable chiefly in the " sweating " of the seams; the surplus will then collect in the bottom of the container where it could be, if desired, drained away. In the case of gas fabric, however, experiments prove (paragraph 11) that this has surprisingly high powers of absorption, so that it is probable that practically none of the water condensed is dissipated into the atmosphere, at least during the rapid changes of state that we are now considering.

[20] Deutsch. Met. Zeit., Vol. III., 1884, p. 421.
[21] Abh. d. K.P. Met. Inst., Vol. III, No. 6, Berlin, 1900.

To estimate the magnitude of these effects combined, it is best to make a numerical calculation for an extreme case. Firstly, it is evident from Graph C that the change of saturated vapour pressure (and, *ipso facto*, of vapour density) per °C. is greatest when the gas temperature is the maximum possible; hence we will consider a container situated initially at ground level with its gas saturated and at a temperature of 45°C. Secondly, other things being equal, the "adiabatic" heating or cooling of helium is greater than of hydrogen, so that the enclosed gas will be assumed to be helium undiluted with air. Suppose now the container be suddenly transported to a height of 1,000ft. Then if both the helium and atmosphere had been dry, the "adiabatic" cooling (absolute) of the former would be 4.4°C., as proved above; owing, however, to the presence of water-vapour and other minor causes, the actual cooling will be somewhat less than this, say 4°C., making the gas temperature now 41°C. But since the gas is still saturated at the upper level, the value of $.46\{100\, r_0\pi_{T+t}/(P+p)\}$ has diminished from 4.4 to 3.5, *i.e.*, the "direct" loss of lift is .9 per cent. We must now add to this the "indirect" loss due to the deposit of moisture, and to fix our ideas we will suppose the volume of the container at ground level to be 100,000 cubic metres or, roughly, 3,530,000 cu. ft., and consequently its lift there to be approximately 100 tons. The saturated vapour pressures of water at 45°C. and 41°C., expressed in mm. of mercury at 0°C., are 72 and 58 respectively, and from the rule given in paragraph 2, these figures also represent the density, in grams per cubic metre, of the vapour in the gas at these two temperatures, and hence also at the two levels considered. On transferring the container from the one position to the other, there will be in general a slight expansion of its volume (about 5 per cent., more or less, according to the atmosphere "lapse rate" and other circumstances), but for the present purpose this can be neglected, especially as by so doing we obviously under-estimate slightly the total amount of water still in vapour form, and correspondingly over-estimate the amount deposited. We thus deduce that each cubic metre of gas in the container must deposit $72-58$ or 14 gr. of moisture during the ascent, from which the total deposited is 14×10^5 gr. or about 1.4 tons. If then the whole of this is retained, the "indirect" loss of lift is 1.4 per cent., making the total loss from both causes $1.4+.9$ or about $2\frac{1}{4}$ per cent.

This is quite considerable, but is evidently the result of extreme conditions in every way. Moreover, in the airship of the future, which will most probably be metal-clad, the amount of gas fabric utilised will be restricted and any "free" water condensed would be drained away or, if necessary, used as ballast. Hence it will be more likely that under most circumstances the lifting gas will be unsaturated, in which case, as shown above, changes of vapour pressure have no effect. For practical purposes, then, the influence of such on the gross lift can be ignored.

Next, consider the term $\frac{1}{2}(100\, r_A\pi_T/930$ mb.), which is, more exactly, $.42\ (100\, r_A\pi_T/P)$ for the case of hydrogen, and $.46\ (100\, r_A\pi_T/P)$ for that of helium. Its value obviously depends upon meteorological conditions external to the container altogether, and in hot climates can vary quite considerably in a short time, such variation being in theory as much as 5 per cent. (Graph E). It is not likely, however, that this extreme alteration would ever be experienced in practice, but nevertheless the effect can still be quite noticeable under likely conditions in the tropics. Suppose, for example, that an airship has been crossing a very hot and dry area such as the Arabian Desert and is about to proceed into the hot but humid Persian Gulf region. In the boundary zone it would be quite possible to be at one moment in air only at 20 per cent. relative humidity, say, and then to move in and out of a cloud where practically complete saturation existed. Even if the temperature of the air were uniform throughout at, say, 40°C. (so that any changes of superheating were negligible), yet the above "humidity" term would suddenly "oscillate" in value from .8 to 3.5 and back again, in-

volving in the case of an airship of similar size to the R.100 or R.101 (and therefore of a displacement in round figures equal to 150 tons) a temporary change of gross lift of about 4 tons. This would be quite appreciable, so that evidently this phenomenon, which is of course practically independent of the nature of the envelope and lifting gas, should be taken into account when planning flights in tropical countries.

On the whole, however, airships operate most frequently in temperate climates where the usual meteorological conditions are such that both the air and gas " humidity " terms can be ignored, and in such cases the only factor which substantially affects the gross lift is the superheating or super-cooling of the gas. Equation (xxxii) thus reduces to:—

" False " lift (or " latent " lift if negative in sign) of gas container very approximately $= (1.16\ t/T) \times$ (" true " lift)

Let us now denote by L and L_0 the gross and " true " lift respectively. Then we have:—

$$L = L_0 \cdot (1 + 1.16\ t/T)$$

whence, by taking logarithms and differentiating

$$dL/L = 1.16\ [(dt/T) - (t\ .\ dT/T^2)]/[1 + 1.16\ t/T]$$

For horizontal movements of the container $dt = -dT$ sensibly, as we have seen, so that the above reduces in this case to:—

$$dL/L = -1.16\ (dT/T) \quad . \quad . \quad . \quad . \quad \text{(xxxv)}$$

neglecting the expression $[1 + t/T]/[1 + 1.16\ t/T]$ or $[1 - .16\ t/T]$, since this is practically equal to unity.

The significance of this equation, in plain language, is that if a container be moved horizontally from one atmospheric zone to another of different temperature (*e.g.*, by passage into a cloud), then its gross lift will decrease with increase of the latter at a rate which is practically independent of any existing superheating, but which becomes greater the lower the atmospheric temperature; conversely for the increase of gross lift when the latter decreases. Numerically we find that when $T = 15°C.$ (say) the percentage change of lift is about .4 per cent. per °C. This loss or gain is, strictly speaking, only temporary, since the virtual superheating or super-cooling will gradually disappear, due to thermal conduction and convection, other conditions remaining the same.

For a small vertical movement dh of the container, $dT = -\sigma\ .\ dh$, where σ is, as hitherto, the " lapse rate " in the atmosphere. If, as is usually the case, this movement is sufficiently rapid to be justified in assuming that the change of state of the lifting gas is practically " adiabatic," then the fall of temperature of the latter is given by equation (xxxiv). Now the general equation for dL/L given above can be thrown into the form:—

$$(1/L)\ (dL/dh) = 1.16\ [\ \{\ d\ (T+t)/dh\ \} - \{\ 1 + t/T\ \}\ \{\ dT/dh\ \}\ ]/T\ [1 + 1.16\ t/T]$$

and, on substituting for dT/dh and $d\ (T+t)/dh$, this reduces to:—

$$(1/L)\ (dL/dh) = -1.16\ [\ \{\ (\gamma - 1)/\gamma\ \}\ \{\ gM/R\ \} - \sigma\ ]/T \quad . \quad \text{(xxxvi)}$$

disregarding the expression $[1 + t/T]/[1 + 1.16\ t/T]$ as above.

We thus have an equation giving the amount of loss of lift with ascent (and conversely gain with descent), such loss or gain being, of course, only temporary, as in the case just discussed. Unlike the corresponding decrease or increase of the absolute temperature of the gas, it is practically independent of any existing superheating or super-cooling, but on the other hand is clearly a maximum, as with the former, when γ and M are greatest, or the gas is pure and dry and the air dry also. The rate is also greater in cold air, and above all when the " lapse rate " σ becomes negative, or when there is an " inversion " in the atmospheric temperature gradient. In this case there is experienced that very stable condition, well-known to pilots of airships, when very considerable dynamic or other

forces have to be employed in order to alter height. Generally speaking, then, we should expect the phenomenon to be most pronounced in the dry and cold air of Polar regions, where "inversions" might conceivably occur also.

The value of $\{(\gamma-1)/\gamma\}\{gM/R\}$ with dry air and for pure and dry hydrogen is 10°C. per km. or 3°C. per 1,000ft., and for pure and dry helium, 13.2°C. per km. or 4°C. per 1,000ft. In an average I.C.A.N. atmosphere, then, where the "lapse rate" is approximately +2°C. per 1,000ft., the maximum value of $[\{(\gamma-1)/\gamma\}\{gM/R\}-\sigma]$ is 1°C. per 1,000ft. for hydrogen, and 2°C. per 1,000ft. for helium. If we take as a good approximation $T=283°$A. (equivalent to the average atmospheric temperature at an altitude of 2,500ft.), then from equation (xxxv) the maximum rate of loss of lift with ascent is about .4 per cent. and .8 per cent. per 1,000ft. respectively for the two gases. On the other hand, if the "lapse rate" were −10°C. per km. or, say, −3°C. per 1,000ft., then with $T=-23°$C. or 250°A. (say) these rates would be 2.8 per cent. and 3.2 per cent.; thus the example rather strikingly demonstrates the reason for the abnormal stability mentioned above.

Lastly, we have to consider the development of a convenient practical formula which will enable the navigator of an airship to determine at any time the total "false" or "latent" lift of the latter, even though the superheating in the various individual gas containers varies widely.

By summation for all the latter in a ship we evidently have, from the approximate equation given above:—

"False" lift (or "latent" lift if negative in sign) of airship very approximately
$$=\Sigma\,(1.16\,t/T)\times(\text{"true" lift})$$
the Σ having the usual significance.

But the "true" lift of any one of the containers is given by equation (xxxi), and in this expression the only term which varies appreciably from one to the other is the "full" volume. Taking the "percentage fulness," for example, this will always in a commercial airship be as near 100 per cent. as possible, from obvious reasons of economy; moreover, in all cases it is desirable that the containers should "blow-off" at about the same "pressure height," so that they will be normally maintained at an approximately uniform degree of fulness. Again the absolute gas temperature, "percentage dry purity" and "percentage gas humidity factor" will not differ widely from one to the other.

With the same notation as paragraph 6, then, we can say that the "true" lift of the individual containers equals wV_1, wV_2, . . . etc., where w is a quantity, which, although not a constant, does not alter sensibly from one to the other; also that the "true" lift of the ship $=wV_1+wV_2+\ldots$ etc. $=wV_o$. Hence, if the individual superheating be denoted by t_1, t_2, . . . etc., we have:—

Gross lift of airship approximately
$$=wV_o+1.16\,\Sigma\,(t_1/T)\,(wV_1)$$
$$=wV_o\,[1+(1.16/T)\,(\Sigma V_1t_1/V_o)]$$

The expression $\Sigma V_1t_1/V_o$ may thus be regarded as the "mean superheating" of the whole mass of lifting gas in an airship, and can be computed once the values of t_1, t_2, . . . etc., have been determined. Again, the air temperature T is, strictly speaking, a variable, but in temperate climates no great error is involved (see Graph F) if it be assumed constant and equal to 283°A., this being the mean atmospheric temperature at a height of 2,500ft. We thus finally deduce that:—

"False" lift (or "latent" lift if negative in sign) of an airship very approximately $=[(.004/°\text{C.})\,(\Sigma V_1t_1/V_o)]\times[\text{"true" lift}]$ (xxxvii)

Under flight conditions, however, the determination of the "mean superheating" is still somewhat tedious if, as in a rigid airship of the conventional type, the number of gas containers therein be large. The writer therefore sug-

gests that the well-known " electrical resistance " gas thermometer could be adapted for the purpose in the following way :—

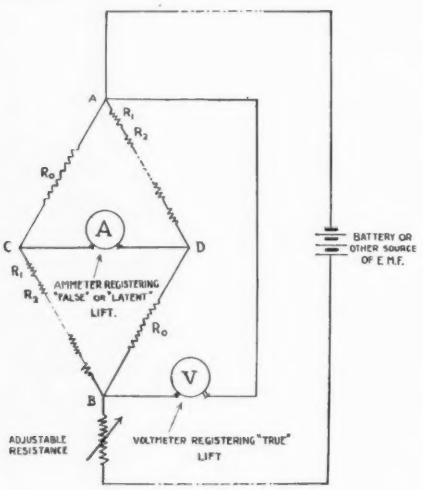


FIG. 5.

Fig. 5 represents a " Wheatstone Bridge " circuit in which the AD and BC " arms " consist of two identical copper or aluminium wires passing through the containers at the height of the central axis of the ship, and hence approximately at the level of the C.B. of each. The length of wire enclosed in each container is adjusted to be proportional to the corresponding " full " volume of the latter, so that if R_1, R_2, . . . etc., be the resistances of the various segments at o°C., then $R_1/V_1 = R_2/V_2 = \ldots$ etc. The AC and BD " arms " consist of two identical coiled wires exposed to the atmosphere outside the ship at the same level as the above, and preferably enclosed in light tubes so as to be shielded from solar or other radiation; their lengths are such that their resistances R_o at o°C. equal $R_1 + R_2 + \ldots$ etc. Hence, if the whole of the lifting gas and the air be at a temperature of o°C. the " bridge " is in balance.

Suppose now that the temperature of the air rises to τ°C. and that of the gas in the containers to $(\tau + t_1)$, $(\tau + t_2)$, . . . etc. Then the resistances now become $R_1\{1 + \alpha(\tau + t_1)\}$, $R_2\{1 + \alpha(\tau + t_2)\}$, . . . etc., and $R_o\{1 + \alpha\tau\}$ respectively, where α is the temperature coefficient of resistance of the material of the wires. It can be easily shown by an application of Kirchhoff's Laws that the current in the " cross arm " CD will now be very nearly proportional to the voltage E, say, applied between A and B and to the difference of the resistances of AD and BC. That is

$$\begin{aligned}\text{Current in } CD &\propto E \times [R_1\{1 + \alpha(\tau + t_1)\} + R_2\{1 + \alpha(\tau + t_2)\} \\ &\quad + \ldots - R_o\{1 + \alpha\tau\}] \\ &\propto E \times [\Sigma R_1 t_1] \text{ since } R_o = R_1 + R_2 + \ldots \text{ etc.} \\ &\propto E \times [\Sigma V_1 t_1] \text{ from the design of the resistances.}\end{aligned}$$

If then the variable resistance shown in the figure be adjusted such that E is proportional to the " true " lift of the airship, then the current in the cross arm CD will be proportional to the " false " or " latent " lift, from equation (xxxv). The voltmeter V and ammeter A shown in the figure can thus be calibrated accordingly.

In practice, it would be preferable to mount the movements of V and A upon a common spindle, in which case, by a suitable choice of the number of turns on the respective coils and by other arrangements well-known to electrical engineers, the combined instrument could easily be designed to indicate gross lift directly, such indications, however, being dependent upon an initial value having been " set " upon the dial after an observation taken, for example, at " pressure height." The readings of the instrument would also, of course, be valid only as long as there were no " valving " from any of the containers and diffusion could be neglected, and it would also be useless above " pressure height " and in cases where the " humidity " effect discussed above were considerable. Even with these limitations, however, there is evidently considerable scope for such an instrument, especially as it is both theoretically and practically preferable that the " resistance coils " should be made of such a cheap and light material as aluminium wire.

(*b*) *Gas Containers Full.*

This condition is, generally speaking, more unusual in practice than the one just discussed, and appropriately enough the variation of the gross lift of an airship under these circumstances can be analysed somewhat more easily. The gross lift is now given by equation (xxvi), and, as already noticed in paragraph 8, is practically unaffected by any changes of the temperature and pressure of the lifting gas, and only to a very minor degree by changes in its humidity. We can, in fact, consider the airship as virtually comprising a single gas container, and, since the " mean percentage dry purity " is sensibly constant, are only concerned with alterations in the temperature and pressure of the air and in the " percentage humidity factor."

This latter is, from equation (xxiii), denoted by

$$[100-\tfrac{1}{2}\{(100\, r_A\pi_T/930 \text{ mb.})+(100\, r_G\pi_{T+t}/930 \text{ mb.})\}],$$

so that the expression

$$\tfrac{1}{2}\{(100\, r_A\pi_T/930 \text{ mb.})+(100\, r_G\pi_{T+t}/930 \text{ mb.})\}$$

represents the " percentage humidity correction " which must be subtracted from the theoretical gross lift which the airship would possess if its lifting gas and the atmosphere were perfectly dry. In a sense, therefore, the latter may be defined as the " true " lift in the present case, although it is not, of course, a constant. Hence :—

" Latent " lift of full gas container due to air and gas humidity
$=[\tfrac{1}{2}\{(100\, r_A\pi_T/930 \text{ mb.})+(100\, r_G\pi_{T+t}/930 \text{ mb.})\}]\times[\text{" true " lift}]/100.$

In comparison, we have from equation (xxxii) :—

" Latent " lift of partially full container due to air and gas humidity
$=[\tfrac{1}{2}\{(100\, r_A\pi_T/930 \text{ mb.})-(100\, r_G\pi_{T+t}/930 \text{ mb.})\}]\times[\text{" true " lift}]/100$

and from the corresponding expressions it will be noticed that any alteration of the term $\tfrac{1}{2}\{(100\, r_A\pi_T/930 \text{ mb.})\}$, representing the effect of vapour pressure in the atmosphere, has an exactly similar effect whether the gas container be full or only partially full. The discussion already given in the last sub-paragraph applies, therefore, equally well to both cases, though it is obvious, however, that in the former one such alterations are generally less in magnitude than in the latter, since an airship would then be above " pressure height " and therefore usually at an altitude where the atmospheric temperature and humidity would be low.

On the other hand, the term $\tfrac{1}{2}\{(100\, r_G\pi_{T+t}/930 \text{ mb.})\}$, representing the effect of vapour pressure in the lifting gas, is of opposite sign in the corresponding expressions above, but nevertheless, if the gas be unsaturated, is a constant in both cases, as shown in the last sub-paragraph. When the gas is saturated, moreover, the " direct " and " indirect " effects there discussed now act one

against the other, and therefore tend mutually to cancel out, since a decrease of $r_0\pi_{T+t}$ increases the gross lift, but at the same time causes deposit of moisture. From the numerical example given for an extreme case, it is evident that this mutual cancellation is almost complete, so that in practice the variation of the above term can always be neglected.

Except under very unusual conditions, then, it may be assumed that the gross lift varies in the present case with the atmospheric pressure and temperature only, so that from equation (xxvi):—

$$\text{Gross lift of airship} = L \text{ (say)} = (\text{constant}) \times (P/T)$$

whence, by taking logarithms and differentiating

$$dL/L = (dP/P) - (dT/T)$$

For horizontal movements of the airship $dP = 0$ sensibly, so that the above reduces to:—

$$dL/L = -(dT/T) \quad . \quad . \quad . \quad . \quad \text{(xxxviii)}$$

This may be compared with the corresponding equation (xxxv) in the "partially full" case, and although any change of lift consequent upon alteration of air temperature is now permanent, it evidently depends upon atmospheric conditions generally in exactly the same way, but is now somewhat reduced in magnitude, *e.g.*, when T equals 15°C. it equals .35 per cent. per °C. instead of .4 per cent.

For a small vertical movement dh, we have, by similar methods to those adopted in developing equations (xxxiv) and (xxxvi):—

$$(1/L)\ (dL/dh) = -[\ \{\ gM/R\ \} - \sigma\ ]/T \quad . \quad . \quad \text{(xxxix)}$$

which again is the analogous expression in the "full" case to equation (xxxvi) in the "partially full" one. The loss of lift with ascent (and conversely gain with descent) is now (as above) permanent and, moreover, independent of the nature of the lifting gas, but nevertheless still increases as the atmosphere becomes drier and colder and when there is an "inversion" of the atmospheric temperature gradient (a combination of circumstances which, it has been pointed out, might easily occur in the Polar regions). The effect of any change of σ is not now, however, so pronounced as in the "partially full" case, since the term $\{\ gM/R\ \}$ is no longer multiplied by the fractional $\{\ (\gamma - 1)/\gamma\ \}$, and in fact equals, when the air is perfectly dry, 34.2°C. per km., or 10.5°C. per 1,000ft.

In an average I.C.A.N. atmosphere, where $\sigma = +2$°C. per 1,000ft. approximately, and when $T = 283$°A., we find that the loss of lift per 1,000ft. of ascent, and conversely gain with descent, is almost exactly 3 per cent., which is, of course, the figure usually employed by airship pilots for rough purposes of navigation.

In concluding this paragraph, it is desirable to make some reference to the physical factors governing the design of automatic valves, and to the effects of super-pressure generally. In fabric-covered ships the latter is rarely permitted to exceed 1in. of water, or say 2.5 mb. (paragraph 6), so that when gas is released on ascent above "pressure height" it may for practical purposes be assumed to expand into the atmosphere whilst keeping always at the same pressure as the latter. Also the increase of altitude will in general be at a rate which makes such expansion practically "adiabatic," and hence governed by a relation of the form given by equation (xxxiii) (*a*). If then we denote by P (as usual) the pressure of the atmosphere, by V_0 the "full" volume of a container, and by dv the volume of gas which is discharged when the former rises a small distance dh in time dt, we have clearly (due regard being paid to sign):—

$$(P + dP)\ (V_0 + dv)^\gamma = P V_0^\gamma$$

whence

$$(dP/P) + \gamma\ (dv/V_0) = 0.$$

As in equations (xxxiv) and (xxxvi), $(dP/P) = -(gM/RT)\ dh$, so that, substituting and transposing, we deduce finally that

$$(1/V_0)\ (dv/dt) = (1/\gamma)\ (gM/R)\ (dh/dt)/T \quad . \quad . \quad . \quad \text{(xl)}$$

This expression gives the rate of discharge of gas with ascent, assuming "adiabatic" conditions. It evidently becomes greater (*a*) the nearer the virtual value of γ approaches its limit unity. As we have seen, this virtual value is reduced whenever there is a departure from strictly "adiabatic" conditions, or when the lifting gas is moist and impure (*b*); the greater the virtual value of M, that is the drier the atmosphere (*c*), the greater the "rate of climb" (dh/dt) of the container and airship (*d*), the lower the value of T or the colder the atmosphere.

It is of very great importance[22] that the automatic valve of a gas container should be dimensioned adequately to cope with the most extreme rate of discharge of gas likely to be expected, for example, when an airship is suddenly forced up in a line-squall. From the above equation it will be seen that the only really safe provision to make is for the worst possible conditions occurring simultaneously, *i.e.*, when $\gamma=1$ approximately, M is for dry air (making $gM/R = 10.5°C.$ per 1,000ft.), (dh/dt) equals 3,000ft. per minute (say), and T is the lowest possible atmospheric temperature, or 220°A., say. The rate of discharge then works out at about 15 per cent. of the "full" volume of the container per minute; in practice, however, it would be probably safe to allow for a maximum of about 10 per cent., considering that the above combination of conditions is, to say the least, extremely improbable.

It appears probable that a future development with metal-clad airships will be the utilisation of much higher super-pressures than those customary hitherto,[23] in order to relieve the hull and frame from severe compressive strains when flying at high speeds. The means usually considered for generating these super-pressures are "scoops" and blowers, both of which have serious disadvantages on the score of aerodynamic drag and weight respectively. The fact must not be ignored, therefore, that adequate super-pressures can be very conveniently generated by simply fitting valves to the ballonets constructed to be opened at will and when the super-pressure exceeds a certain safe maximum. Practically any value desired could thus be generated by simply closing or partially closing these valves and increasing height by aerodynamic means or discharge of ballast; in a normal atmosphere the fall of pressure with height would create in an ideal hermetically-sealed container a super-pressure which increases at the rate of about 30 mb. per 1,000ft.

10. *Prediction of "Pressure-Height"*

For many purposes it is very desirable that an airship pilot should be able to predict his probable "mean pressure height" before setting out on a flight, given the initial atmospheric and other conditions, and the gas containers of his ship having been filled to a known (more or less) degree. Such knowledge is usually required in order to strike a balance between the maximum load which it is permissible to carry, and the necessity of attaining a certain height to pass over, say, a range of hills on the route traversed.

A very handy formula for this purpose can be deduced quite readily as follows:—

Denote by the subscript $_H$ all quantities which refer to "pressure height." Then since the mass of pure hydrogen (or helium) in a container is constant at all altitudes up to the latter, we have, as in the derivation of equation (xxx) (*a*), and with the notation as there and elsewhere, for hydrogen

$$\begin{aligned}
&[\lambda V_0 \times \{\, 2.016\, K\,(P+p)\,\}/R\,(T+t)]\,[1-r_G\pi_{T+t}/P] \\
&=[V_0 \times \{\, 2.016\, K\,(P+p)\,\}/R\,(T+t)]\,[1-r_G\pi_{T+t}/P] \\
\text{or}\quad &100\,\lambda = 100\times[(P+p)_H/(P+p)]\times[(T+t)/(T+t)_H] \\
&\quad\times[\,\{1-(r_G\pi_{T+t}/P)\}_H/\{1-(r_G\pi_{T+t}/P)\}\,] \qquad\qquad \text{(xli)}
\end{aligned}$$

[22] Cf. The Report of the Court of Inquiry on the disaster to the U.S. airship "Shenandoah."
[23] Vide the Paper on "The Metalclad Airship" and subsequent discussion.

since K, the " dry purity," is a constant. An identical equation holds for helium.

This equation is an accurate one enabling one to find, if so desired, a previous value of the " percentage fulness " from an observation of " pressure height," and of the conditions on both occasions. It is obviously useless, however, as it stands, for the main purpose in view, simply because it is impossible to predict what the superheating of the gas will be at " pressure height " or, strictly speaking, the humidity conditions there, although these latter are of entirely secondary importance.

The equation can nevertheless be easily simplified if allowance be made for the facts that the prediction of " pressure height " need not be other than approximate. This being the case, we can make the assumptions in equation (xli) that the pressures and temperatures of the gas at both altitudes are the same as those of the air, and that the former obey the relations already given for an I.C.A.N. " standard atmosphere," this latter representing, as it does, a very fair average of atmospheric conditions, especially at high altitudes.

Let now h denote the altitude, as registered by an " uncorrected I.C.A.N." altimeter, of a gas container in an airship when the latter is either at its point of departure, or alternatively at some position where λ for the container has been observed; similarly, let H denote the corresponding " pressure height " for the latter. Then, with the assumption already made, we find, from equation (xxxiv) that

$$100\,\lambda = 100\,[1013.2\,\{1 - .006879\,(H/1{,}000\text{ft.})\}^{5.256}\text{ mb.}]/[1013.2\,\{1 - 006879\,(h/1{,}000\text{ft.})\}^{5.256}\text{ mb.}]\,[288\,\{1 - .006879\,(h/1{,}000\text{ft.})\}\,°\text{A.}]/[288\,\{1 - .006879\,(H/1{,}000\text{ft.})\}\,°\text{A.}]$$

which reduces, since .006879 (h/1,000ft.) and .006879 (H/1,000ft.) are very small relative to unity, to

$$\text{" Percentage fulness "} = 100\,\{1 - .006879\,[(H-h)/1{,}000\text{ft.}]^{5.256}\} = 100 - 2.93\,([H-h]/1{,}000\text{ft.}) \text{ for moderate altitudes.}$$

Putting $(H-h)=h_1$, the " difference in height," we can now plot the above expression, as has been done in Graph G, and from the latter deduce the " pressure height " corresponding to any initial " percentage fulness," or vice-versa.

If the container were at " mean sea-level," then $h=0$ and $h_1=H$; the graph then gives " percentage fulness at mean sea-level " in terms of stipulated " pressure height." Finally it can be used in an obvious manner to determine " mean pressure height," if the " mean percentage fulness " of the gas container of an airship has been estimated. Thus, to take an example, if the point of departure of the ship be at height 200ft. and her " ceiling " is required to be 5,000ft., then $H-h=4{,}800$ft.; the gas containers must then be filled to a " mean percentage fulness " of approximately 85 per cent.

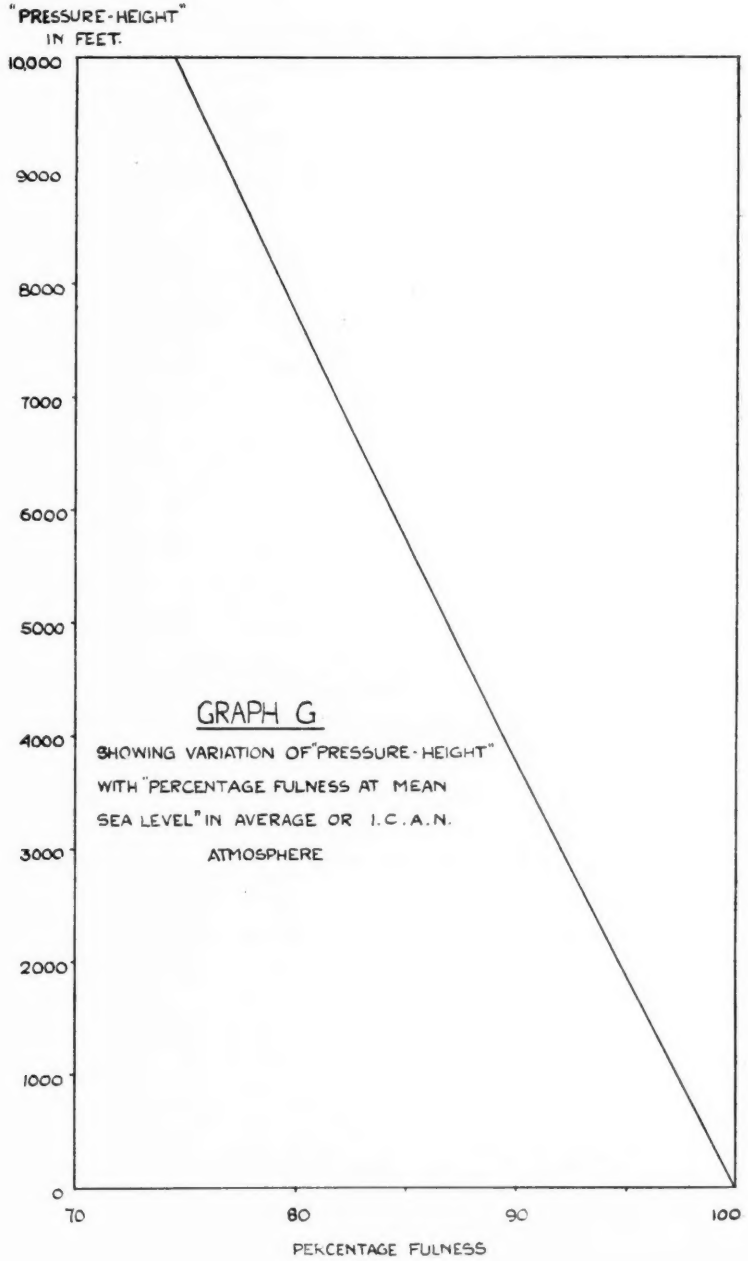
In paragraph 6 it is mentioned that an alternative definition of " percentage fulness " is sometimes given. This is that the latter is the ratio of the density of the gas at " pressure height " to that which it has when the fulness is measured. This is not quite correct, for to use the notation at the beginning of paragraph 9, if ρ_{GH} denotes the " partial " density of the hydrogen (or helium) in the container, ρ_{GA} that of the air therein, and ρ_{GW} that of the water-vapour, then

$$\text{Density of gas} = \rho_{GH} + \rho_{GA} + \rho_{GW}$$

Now all that can be said is that, neglecting diffusion and presuming no valving, the masses both of hydrogen (or helium) and air in the container remain constant up to " pressure height." This is not necessarily the case, however, with the water vapour, which can, and does, lose mass by condensation.

Hence,

$$\lambda V_0 \times (\rho_{GH} + \rho_{GA}) = V_0 \times (\rho_{GH} + \rho_{GA})_H$$
$$\text{or } \lambda = (\rho_{GH} + \rho_{GA})_H / (\rho_{GH} + \rho_{GA})$$



GRAPH G

SHOWING VARIATION OF "PRESSURE-HEIGHT" WITH "PERCENTAGE FULNESS AT MEAN SEA LEVEL" IN AVERAGE OR I.C.A.N. ATMOSPHERE

The other definition would give, however,

$$\lambda = (\rho_{GH} + \rho_{GA} + \rho_{GW})_H / (\rho_{GH} + \rho_{GA} + \rho_{GW})$$

and the two are not quite synonymous except when the gas is dry.

11. *Soakage of Fabric*

As mentioned in the introduction, in parallel with the more theoretical investigations the results of which have been given in the previous paragraphs, a series of experiments was undertaken by the writer and others to determine the extent to which both the outer cover and the gasbags of a fabric-covered airship could absorb moisture under various conditions of temperature and humidity. For this purpose a double-walled cupboard was prepared in the form roughly of a cube of side 3ft., the interspace between the walls being well " lagged " with asbestos wool. Heating of the interior was effected by means of a series of electric lamps with output controllable by an external rheostat and arrangement of switches, also any desired degree of humidity could be transmitted to the contained air from water in shallow trays, the latter being arranged so that they could be insulated from the main compartment, when necessary, by airtight panels.

Measurement of the internal temperature was made by a delicate mercury thermometer and of the relative humidity by a " Lambrecht polymeter " or " hair hygrometer," these instruments being mounted inside the cupboard and observed through a small window fitted in one of the sides of the latter.

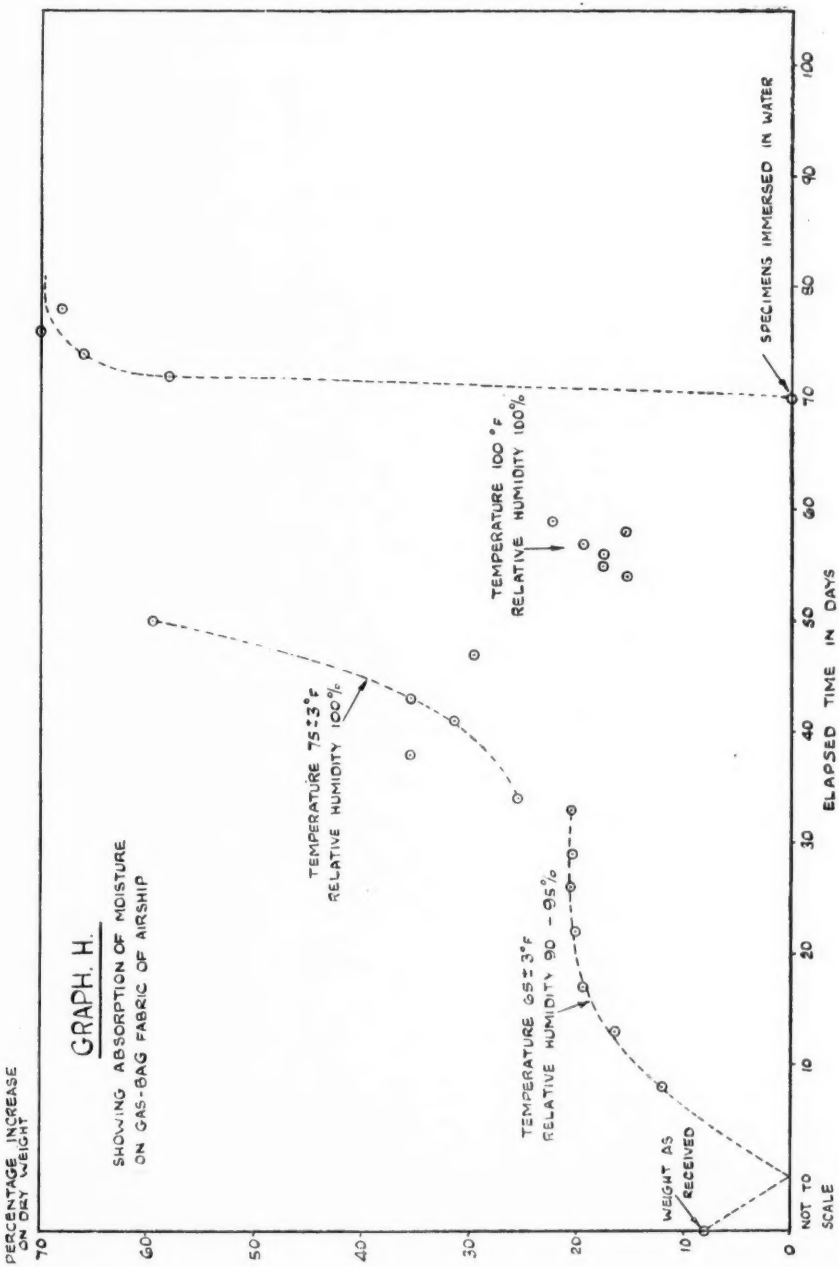
The resources and time available did not permit of the installation of any apparatus for automatically maintaining the temperature and relative humidity constant, but it was found in practice that due to the efficiency of the " lagging " arrangements the former did not vary appreciably once adjustment had been made on the rheostat, and comparatively little personal attention was required. The relative humidity could also be kept reasonably constant, especially as all the experiments were for obvious reasons made with the atmosphere in the cupboard at or near saturation point.

The standard procedure adopted was to take a number of samples of equal area from the fabric to be tested, weigh them carefully on a delicate chemical balance, and then thoroughly dry them in a desiccator. After re-weighing, they were then placed in the cupboard and the conditions therein adjusted to a certain temperature and relative humidity. At various intervals afterwards, usually every day, these weighings were repeated, care being taken in the handling to disturb as little as possible of the absorbed moisture. Finally the results so obtained were meaned and " plotted " against time, the basis being percentage increase of weight on that of the dry state.

It will be convenient to deal with the results under separate headings as below :—

(a) Gas-Bag Fabric

The samples in this case consisted of four pieces of fabric, each of area 1 sq. ft., cut from a roll recently received, and similar (although very slightly heavier) to that used in the airship R.100. The procedure adopted being as above, the results are seen from Graph H. For the first 33 days the applied relative humidity ranged from 90-95 per cent. and the temperature was $65 \pm 3°F$. As will be noticed, the increase of weight on the " dry " state was after this time only about 20 per cent. and had already become stabilised. The temperature was then raised to $75 \pm 3°F$., and the relative humidity to 100 per cent., with the result that the absorption increased at a very rapid rate, and 18 days later had reached a degree such that the increase of weight had now become 60 per cent., and showed no signs of having reached a limit. It was then decided to stimulate as far as possible conditions in a tropical and very humid climate, and for this


GRAPH. H.
SHOWING ABSORPTION OF MOISTURE
ON GAS-BAG FABRIC OF AIRSHIP
PERCENTAGE INCREASE ON DRY WEIGHT
ELAPSED TIME IN DAYS
NOT TO SCALE
WEIGHT AS RECEIVED
TEMPERATURE 65±3°F RELATIVE HUMIDITY 90 - 95%
TEMPERATURE 75±3°F RELATIVE HUMIDITY 100%
TEMPERATURE 100°F RELATIVE HUMIDITY 100%
SPECIMENS IMMERSED IN WATER
0
10
20
30
40
50
60
70
80
90
100

purpose the temperature in the cupboard was further increased to 100°F., the relative humidity being kept as before at 100 per cent. Somewhat contrary to expectations it was now found that the absorption decreased in a very marked manner, and within a day the increase of weight had become 20 per cent. only, thereafter remaining practically constant under these conditions. Finally the specimens were completely immersed in water at room temperature (about 70°F.) and the soaking continued for 6 days; at the end of that time the increase of weight appeared to be stabilised at about 70 per cent., which is thus presumably the maximum possible " soakage."

The writer suggests the following very rough theory to account for the above facts, and also to explain the phenomenally large diffusion of water-vapour through gas fabric.

Since the gold-beaters skins on the inside of a gas-bag consist of animal membranes, they may be assumed to have the form of very thin diaphragms percolated by a large number of very small holes, the latter on the average being shaped somewhat like a dice-box.

Hence a piece of the membrane will show, when highly magnified, an appearance something like Fig. 6 (*a*). Now in general in each of the cells there will be a small globule of water held in place chiefly by surface-tension, and the surfaces of this on both sides will have a curvature given by $p = 2TR$,[24] where p is the external vapour pressure on the particular side considered, T is the surface tension of water, and R is the radius of curvature of the surface. This assumes that the weight of the globule is negligible in comparison with the other forces acting on it, which may be accepted. Now let us consider firstly the case when the gases on both sides of the fabric are unsaturated but at equal vapour pressures. Then since the face of the membrane inside a hole will be, generally speaking, dry except where in contact with the globules, the surfaces of the latter will meet it at a definite " angle of contact." Hence for the appropriate curvature the globule must have a form such as $A\ B\ C\ D$ (Fig. 6 (*b*)). If now the temperature of the adjacent gases be lowered, keeping, however, their vapour pressure the same, then at saturation there will be films of moisture deposited all over the face, which is therefore " wetted." The " angle of contact " now becomes zero, so that although the curvature of the globular surfaces remains the same, the latter, to preserve equilibrium, move to $A_1\ B_1\ C_1\ D_1$ where they become tangential to the face. Clearly the volume comprised by $A_1\ B_1\ C_1\ D_1$ is much greater than $A\ B\ C\ D$, so that the sudden absorption of moisture at saturation is explained. If now the vapour pressures, whilst still saturated, be increased through rise of temperature the curvature of the globular surfaces must increase, so that the latter, although still tangential to the face, recede to $E\ F\ G\ H$. This involves a diminution of volume, so that with rise of temperature the absorption is less, as is found by experiment.

Next consider the more general case where the vapour-pressures on the two sides are unequal. Then similar considerations to the above apply except that the curvatures of the globular surfaces $L\ M$ and $N\ P$ will now differ (Fig. 6 (*c*)), that $N\ P$ facing the lower pressure being flatter than the other $L\ M$. But it is well known that molecules can escape from a liquid surface in greater numbers the less concave this is, so that they will leave $N\ P$ at a greater rate than $L\ M$. On the other hand, molecules will be absorbed by $L\ M$ more rapidly than by $N\ P$ since the adjacent vapour pressure, and hence the number striking the surface in unit time, is greater. There will then be a net diffusion from the direction of higher vapour pressure to that of lower, which is in accordance with experience. Lastly, because of the water " seal," hydrogen (or helium) molecules would pass through only with difficulty.

The above phenomena are of considerable importance practically, for it definitely appears that whilst voyaging in very hot and humid areas an airship

[24] See, for example, Poynting and Thomson's " Properties of Matter," Chap. XIV.

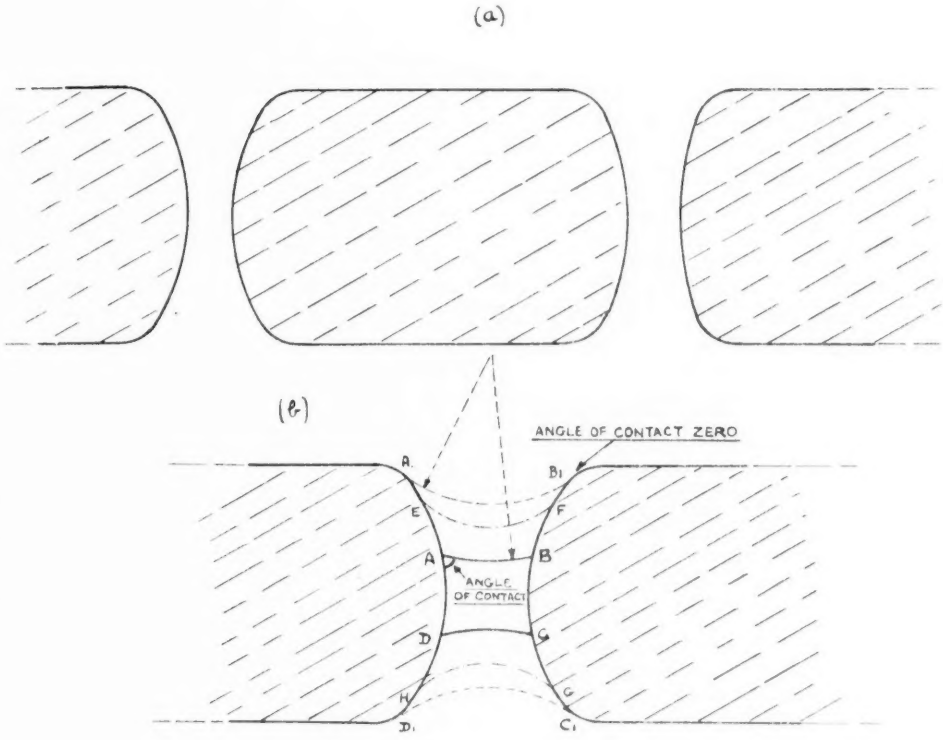

(a)
(b)
ANGLE OF CONTACT ZERO
A
B1
E
F
A
B
ANGLE OF CONTACT
D
G
H
G
D1
C1
(c)
HIGHER VAPOUR - PRESSURE
L
N1
P
N
LOWER VAPOUR - PRESSURE

FIG. 6.

would experience little increase of weight due to any absorption of moisture by her gas fabric. This is somewhat reassuring in connection with any projected England-to-India or England-to-South Africa Airship Service. On the other hand, such absorption can be very serious in temperate climates, and explains in large measure the excess of fixed weights shown by the R.100 on the first " Lift and Trim " trial (paragraph 8). For by the very nature of the processes used in this country commercially for " gassing " airships with hydrogen, it is inevitable that large quantities of moisture should be carried into the gas-bags unless means are provided for partially drying the gas, and this does not appear to be usually the case. Indeed it is a matter of common observation that during such operations the gas mains and bags are usually quite " dripping " wet, which means that the hydrogen is almost certainly saturated. If in addition, then, the atmospheric temperature at the time is fairly low, and the humidity also near saturation, the conditions are entirely conducive to large absorption of moisture.

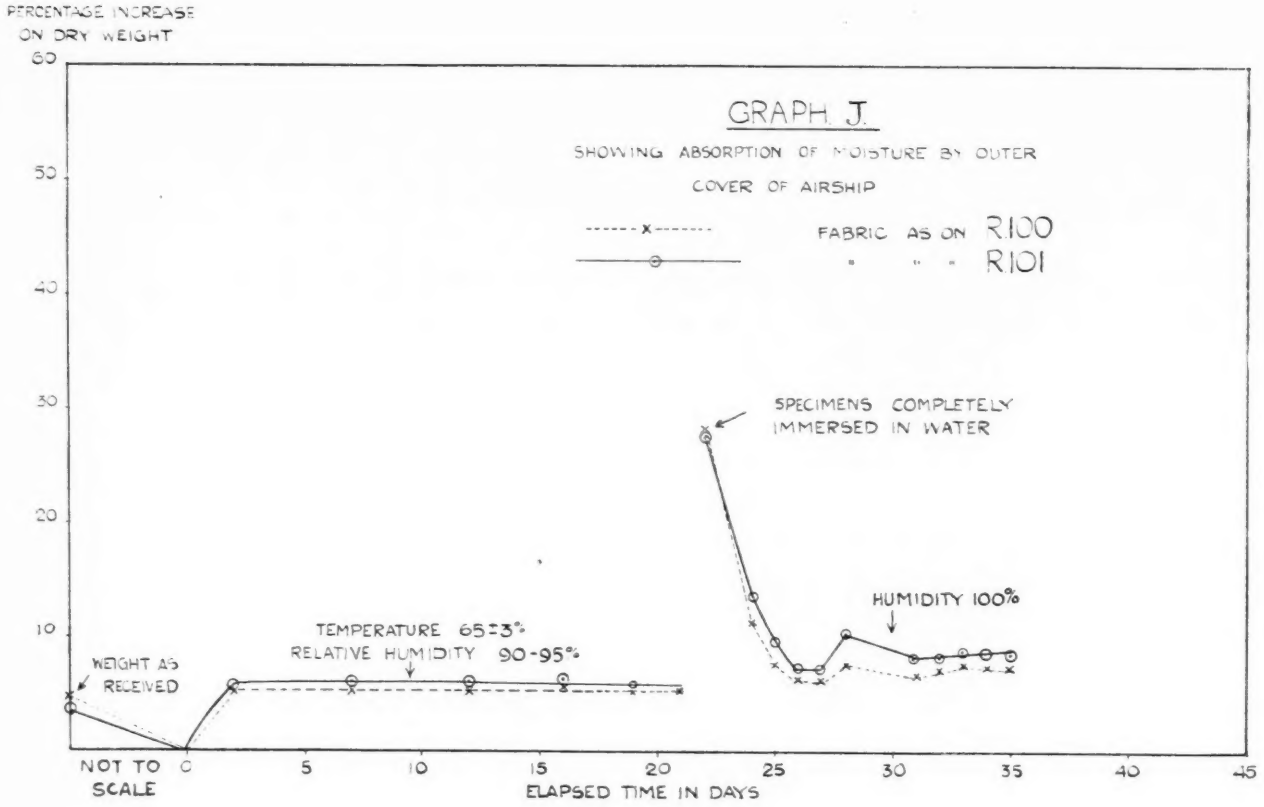
Such was the case at the time of the first " Lift and Trim " trial of the R.100, the data for which are given in paragraph 8, for the records show that " gassing " had been continued up to just before the trial in order to fill all bags to " blowing-off " point and in addition the temperature at the time was low and the atmospheric humidity nearly 100 per cent. The scheduled weight of the gas-bags of the airship is 7.3 tons, so if, in accordance with Graph H an extra 70 per cent. is allowed for soakage, then the increase of weight due to the latter is equal to 5.1 tons. Comparing this with the figure of 5.4 tons given for the discrepancy already mentioned, it will be seen that the latter is very nearly accounted for.

During the five months which elapsed between this trial and the next, the ship frequently received fresh charges of hydrogen before proceeding on flights, etc., and in addition, being winter-time, the atmosphere was generally very humid and cold. It is likely, therefore, that the gas in the bags remained saturated or nearly so and hence the moisture originally absorbed in the " fabric " was retained or even slightly augmented.

Clearly a partial remedy for this state of affairs is to make arrangements to dry the gas to some extent before passing it into the bags. To effect this completely would require large and expensive plant, and that is neither necessary nor desirable, firstly because the experiments prove that the absorption is comparatively small as long as the gas is just not saturated, and secondly because complete dryness would cause bad cracking of the gold-beaters skins. Even so, it is still difficult to prevent the gas becoming saturated whilst the ship is in flight at high altitude, and the low temperatures thereat are, as we have seen, very conducive to large absorption. Clearly, then, it is advisable when designing an airship to allow a good margin, say 50 per cent., on the weight of the gas-bags as received from the makers, and incidentally this should obviously be taken into account when comparing the relative advantages of a fabric and metal-clad airship.

(*b*) *Outer Cover*

In this connection the first experiments were made upon four samples of doped fabric, each 1 sq. ft. in area, similar to that used in the R.100 and upon two samples of fabric such as used in the R.101. These were first exposed for three weeks to a humidity of 90-95 per cent., at a temperature of $65 \pm 3°$, and, as will be seen from Graph J, after two days the increase of weight became in all cases stabilised in the vicinity of 6 per cent. above the " dry " state, or about 1 per cent. above that as received. The specimens were then completely immersed in water for 48 hours, and after that time had increased their weight about 28 per cent. On being re-exposed to the air of the cupboard, however, even though the latter was at 100 per cent. humidity and its temperature was varied to some extent, the specimens within a comparatively short time lost this large excess, and their


PERCENTAGE INCREASE ON DRY WEIGHT
60
50
40
30
20
10
GRAPH J.
SHOWING ABSORPTION OF MOISTURE BY OUTER COVER OF AIRSHIP
FABRIC AS ON R100
" " " R101
SPECIMENS COMPLETELY IMMERSED IN WATER
HUMIDITY 100%
TEMPERATURE 65±3°
RELATIVE HUMIDITY 90-95%
WEIGHT AS RECEIVED
NOT TO SCALE
0
5
10
15
20
25
30
35
40
45
ELAPSED TIME IN DAYS

weight returned to a value uniformly 8 per cent. higher than that of the dry state.

Since the design-weight of the outer cover of the R.100 was given as 4·35 tons, the above experiments prove that even exceptional soaking corresponding to a very heavy rain storm could not increase the weight of the ship in this respect by more than about a ton. This fact also has some bearing with reference to the disaster to R.101, for it tends to show that the "intermittent rain"[25] which she experienced during her final journey, even if it had been much more intense, would have been ineffective in causing any serious increase of weight through soakage in her outer cover.

Upon the completion of the observations described above, another series was commenced, this time with the object of determining the relative advantage with regard to absorption of various textiles and treatments thereon. For this purpose three specimens each of "B" cotton, "C" cotton, and "4 F 1" linen were prepared, the first of which from each fabric was simply dried and exposed to moisture whilst still undoped. The second was dried, ironed and then doped, and the third doped before drying. From Graph K, it will be seen that the general deductions to be drawn from the experiment are that: (*a*) there is no particular advantage in this respect to be gained by drying before doping, at least at all commensurable with the trouble involved in such a procedure, but doping itself is highly efficacious in preventing absorption; (*b*) the behaviour of linen and cotton is practically identical (*c*) as with gas-fabric, high temperatures, even when combined with a saturated atmosphere, tend to reduce the absorption.

Summarising the whole series of experiments, the main conclusions are that gas-fabric is a great offender as regards absorption of moisture if the temperature be low or moderate, but not if the latter be high, and that the soakage in the outer cover is surprisingly small under all circumstances.

12. *Dynamic Effect of Rain*

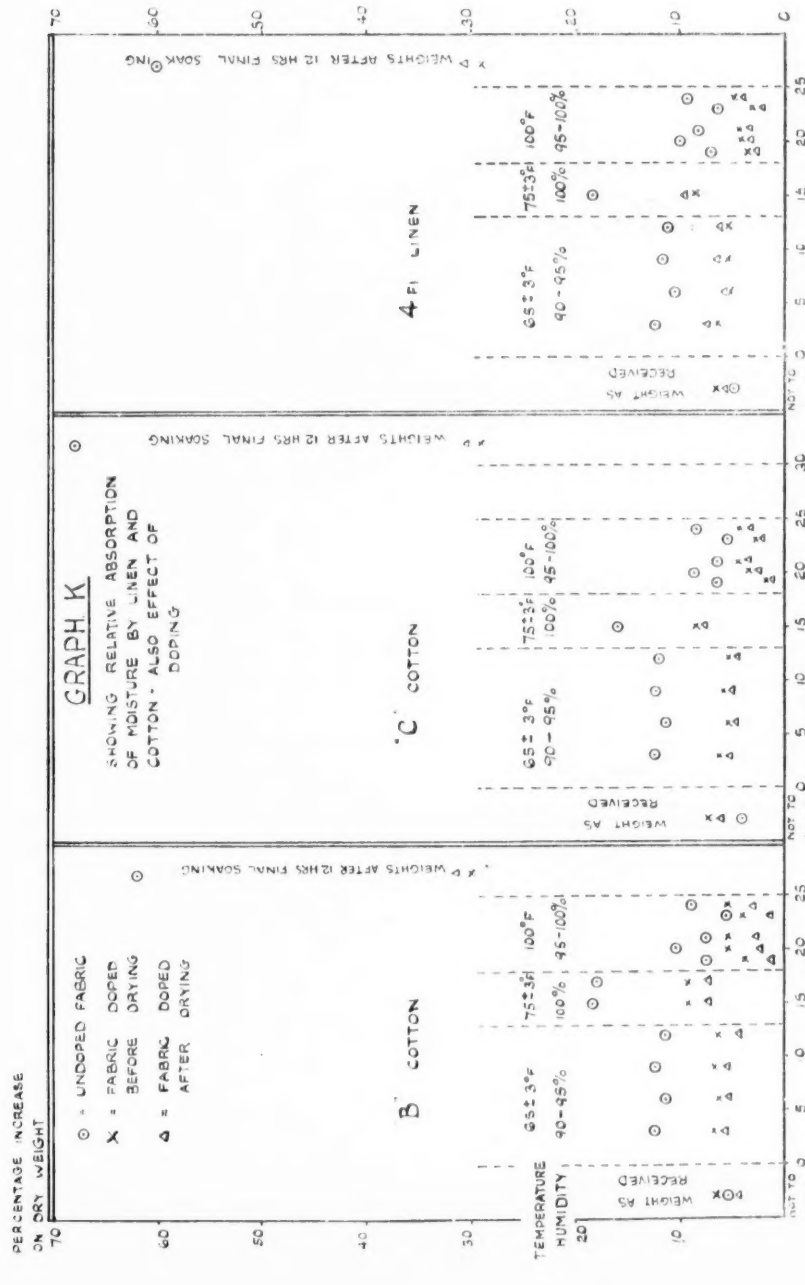
Finally, there is one other effect which should be investigated in any complete analysis of the influence of moisture upon the weight of an airship and its static lift, and that is the downward pressure exerted upon the hull due both to the dynamic impact of rain-drops and the weight of "free" water on its upper surface, this latter being distinct from the "soakage" already investigated.

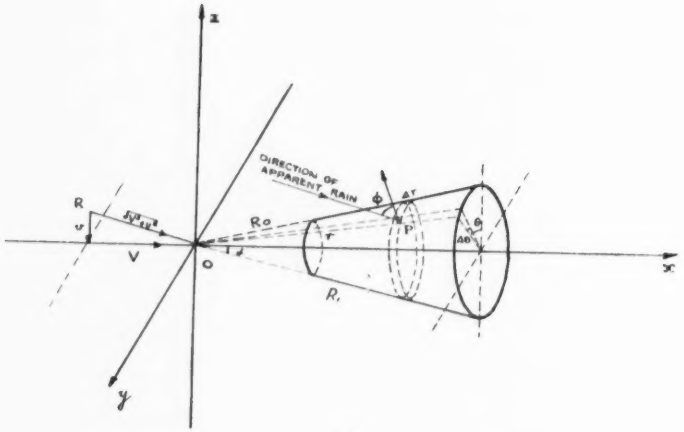
The mathematical treatment of the problem becomes very difficult and tedious if a general solution of the first part is attempted for a ship of any shape whatsoever, but such is hardly necessary since rough numerical values only are required, and in any case the available data with regard to the rate of precipitation of rain are only very approximate. We will attack this problem, then, by considering first of all the pressure on the slant surface of a truncated cone travelling horizontally in the direction of its axis with velocity V and passing through a rain-cloud in which there are n drops per unit volume, each of mass m descending with velocity v. The drops will be assumed to behave like small shot or rigid particles striking a surface, with the exception, however, that the "coefficient of restitution" is taken as zero, it being a matter of common observation that such drops shatter on impact and do not appreciably rebound.

By the principle of relativity the conditions of the problem are obviously not altered if we consider the cone as at rest, and superimpose upon the rain-drops a horizontal velocity V in the opposite direction to that in which this was travelling.

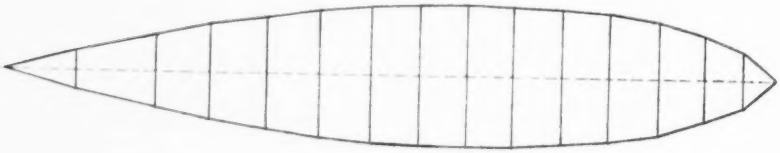
Take axes Ox, Oy and Oz, as shown in Fig. 7 (*a*), and let the semi-angle of the cone be α; also let P be any point on the surface of the latter distant r from O, and denote by θ the angle which the plane OPx makes with Oxz.

[25] Cf. "Report of the R.101 Inquiry," Appendix 5.

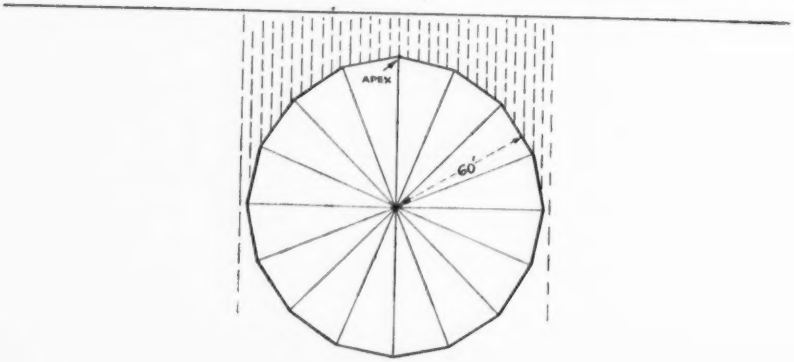

GRAPH K
SHOWING RELATIVE ABSORPTION OF MOISTURE BY LINEN AND COTTON - ALSO EFFECT OF DOPING
⊙ = UNDOPED FABRIC
X = FABRIC DOPED BEFORE DRYING
Δ = FABRIC DOPED AFTER DRYING
PERCENTAGE INCREASE ON DRY WEIGHT
'B' COTTON
'C' COTTON
4Fl LINEN
WEIGHTS AFTER 12 HRS FINAL SOAKING
WEIGHT AS RECEIVED
TEMPERATURE
HUMIDITY
65 ± 3°F
90 - 95%
75 ± 3°F
100%
100°F
95 - 100%
ELAPSED TIME IN DAYS
NOT TO SCALE



(a)



(b)



(c)

FIG. 7.

The Cartesian equation of the cone is clearly

$$(y^2+z^2)/x^2=\tan^2\alpha \text{ or } -x^2\tan^2\alpha+y^2+z^2=0$$

so that the direction cosines of the normal at P are given by

$$-x\tan^2\alpha/(x^2\tan^4\alpha+y^2+z^2)^{\frac{1}{2}},\ y/(x^2\tan^4\alpha+y^2+z^2)^{\frac{1}{2}},\ z/(x^2\tan^4\alpha+y^2+z^2)^{\frac{1}{2}},$$

which, on substituting

$$x=r\cos\alpha,\ y=r\sin\alpha\cos\theta,\ z=r\sin\alpha\cos\theta$$

becomes

$$-\sin\alpha,\ \cos\alpha\sin\theta,\ \cos\alpha\cos\theta$$

The direction of the apparent rain is parallel to RO and the direction cosines of OR are clearly

$$-V/(V^2+v^2)^{\frac{1}{2}},\ 0,\ v/(V^2+v^2)^{\frac{1}{2}}.$$

Hence, the angle ϕ which the apparent rain makes with the normal at P is given by

$$\begin{aligned}\cos\phi&=\{V/(V^2+v^2)^{\frac{1}{2}}\}\sin\alpha+\{v/(V^2+v^2)^{\frac{1}{2}}\}\cos\alpha\cos\theta\\&=\{1/(V^2+v^2)^{\frac{1}{2}}\}(V\sin\alpha+v\cos\alpha\cos\theta).\end{aligned}$$

The angle ϕ becomes greater than 90°, *i.e.*, the rain never actually meets P at all, or the latter is "in shelter," if $(V\sin\alpha+v\cos\alpha\cos\theta)$ is $=0$ or <0, from which we find that the critical angle θ_1, where the rain just grazes the surface is given by

$$\cos\theta_1=-(V/v)\tan\alpha.$$

Hence, it may readily be deduced, and is also obvious from the figure, that (*a*) if $V=0$ then $\theta_1=\pi/2$, (*b*) if $V/v>\cot\alpha$, then θ_1 is imaginary, so that all points on the cone are swept by the rain, (*c*) if V is $=Vc$ and $V/v>\cot\alpha$ numerically then no points are touched by the latter.

Let us now isolate a small surface on the cone adjacent to P by the intersection of two planes given by $x=(r-\frac{1}{2}\Delta r)\cos\alpha$ and $x=(r+\frac{1}{2}\Delta r)\cos\alpha$, and two others given by $\theta+\frac{1}{2}\Delta\theta$ and $\theta-\frac{1}{2}\Delta\theta$. This small surface will be of area $r\sin\alpha\Delta\theta\Delta r$ and its projection perpendicular to OR is $r\sin\alpha\cos\phi\Delta\theta\Delta r$.

Hence, a number of raindrops given by

$$n\times(V^2+v^2)^{\frac{1}{2}}\times r\sin\alpha\cos\phi\Delta\theta\Delta r\Delta t$$

strike the surface in the time Δt, each bringing with it momentum in the direction normal to P equal to

$$m\times(V^2+v^2)^{\frac{1}{2}}\times\cos\phi.$$

Since also these drops do not rebound appreciably, then the total change of momentum in the direction normal to P in time Δt is

$$nm(V^2+v^2)\cos^2\phi\, r\sin\alpha\Delta\theta\Delta r\Delta t$$

From this, by Newton's Second Law, the force on the elementary area normal to the surface and due to the impacts is

$$nm(V^2+v^2)\cos^2\phi\, r\sin\alpha\Delta\theta\Delta r$$

and the component of this vertically downwards is

$$nm(V^2+v^2)\cos^2\phi\, r\sin\alpha\cos\alpha\cos\theta\Delta\theta\Delta r.$$

Hence, for a zone of the cone comprised between the planes $x=R_1\cos\alpha$ and $x=R_0\cos\alpha$ the total vertical force due to the impacts of the raindrops is

$$2nm\sin\alpha\cos\alpha(V^2+v^2)\int_0^{\theta_1}\int_{R_0}^{R_1} r\cos^2\phi\cos\theta d\theta dr$$

$$=\tfrac{1}{2}nm\sin 2\alpha(R_1^2-R_0^2)\int_0^{\theta_1}(V\sin\alpha+v\cos\alpha\cos\theta)^2\cos\theta d\theta$$

Substituting for $\cos\phi$.

But since nmv is the mass of water which is precipitated on unit area of horizontal surface in unit time, we have $nmv/\rho=\delta$ or $nm=\delta\rho/v$ where δ is the precipitation (*e.g.*, in inches per hour) and ρ is the density of water. Hence, the total vertical force due to the impacts:—

$$=\tfrac{1}{2}(\delta\rho/v)\sin 2\alpha\,(R_1^2-R_0^2)\int_0^{\theta_1}(V\sin\alpha+v\cos\alpha\cos\theta)^2\cos\theta\,d\theta.$$

Although the evaluation of the above integral is quite straightforward, the substitution of limits in general produces a somewhat complicated expression and, therefore, we will simply consider the particular cases which will be found to be of the most importance in practice.

If the cone be at rest, so that $V=0$ then $\theta_1=\pi/2$ and the above expression reduces to $4\delta\rho vbh^3/3l^2$, where b is the mean radius of the truncated cone, h is its perpendicular height and $l\,(=R_1-R_0)$ its slant length. As the semi-angle of the cone becomes smaller and smaller l becomes more and more nearly equal to h, so that in the limiting case when the cone has degenerated into a cylinder, the expression becomes $(4/3)\,\delta\rho vbh$. This latter formula also applies to a cylinder moving with any horizontal velocity, since in this case it is clear that the magnitude of the latter makes no difference to the vertical force.

If the cone be moving at high speed, such that $V/v>\cot\alpha$, then for practical purposes $\theta_1=\pi$, the above expression becomes $2\pi\delta\rho Vbh^2d/l^2$ where d is the difference of radius of the plane faces and the other symbols have the same significance as before.

These formulas can now be applied in practical investigations by assuming that the body of an airship is, with sufficient accuracy for the present purpose, in the form of a series of coaxial truncated cones (Fig. 6) (*b*); for convenience, in the case of a rigid airship, the plan boundaries of these are taken as coincident with the transverse frames, complete data concerning the dimensions of which exist for design purposes. If the ship be considered at rest, it is now an easy matter by repeated application of the first formula and summation, to determine the downward force on it due to the heaviest rain storm, the precipitation values being taken from the following table, extracted from W. J. Humphreys' "Physics of the Air" (p. 268):—

Popular Name.	Precipitation intensity in mm. per hour.	Diameter of drop in mm.	Velocity of fall metres per sec.
Clear	0.00	—	—
Fog	Trace	0.01	0.003
Mist	0.05	0.1	0.25
Drizzle	0.25	0.2	0.75
Light rain	1.00	0.45	2.0
Moderate rain ...	4.00	1.0	4.0
Heavy rain	15.00	1.5	5.0
Excessive rain ...	40.00	2.1	6.0
Cloudburst	100.00	3.0	7.0
		5.0	8.0

If, however, the ship be considered as moving at high speed, then the other formula must be used, and the profile "adjusted" slightly in order that all the constituent truncated cones may have either a semi-angle which is zero or such that $V/v>\cot\alpha$ numerically. This is slightly artificial, but leads to an error which can be ignored in practice.

On the above lines a calculation has been made of the effect of a cloudburst on an airship of such a size and shape as the R.100 or R.101. The details need not be given here, but the result shows that when such a ship is at rest (*e.g.*, at a mooring-mast) and exposed to this sort of storm, during which time the precipitation may be as much as 4in. an hour and the average velocity of the rain-

drops about 18 m.p.h. (see previous table), then the downward force due to the impact of the latter is about 1/15 ton; if, however, the ship be moving at a speed of 80 m.p.h. the force works out at about 1/100 ton. Clearly, therefore, this direct effect of rain is completely negligible even in the extreme case of a cloudburst. On the other hand, the impacts from hailstones might be more serious owing to their larger mass and velocity; but even so, the magnitude of the effect is such that it is difficult to believe that they could ever be a serious danger to an airship except, perhaps, by causing damage to the cover, or collecting and congealing in large masses thereon.[26]

Turning next to the other influence of rainfall on lift, *i.e.*, causing collection of " free " water on the upper surface of the cover, this is far more difficult to estimate, it being almost hopeless to try to make any calculations without introducing assumptions so artificial as to make them nearly valueless. The essential difficulties in solving the problem as to how quickly rain runs off the hull lie in the lack of symmetry of the latter about a vertical axis, the unknown roughness of the cover, and the fact that a cross-section of the ship may be, as in the case of the R.100, " fluted."

The writer has, however, attempted a very rough solution for the case of the latter ship or the R.101 on the following lines:—

Consider a 16-sided smooth regular polygon (Fig. 6) (*c*), the radius of whose circumscribing circle is 60ft., and suppose a particle, initially at the apex, is allowed to slide under the force of gravity down the sides and to drop off. It is easy to prove that the time what it will take to do so is about $\frac{1}{4}$-minute, and consequently the average of such times for particles placed anywhere on the upper surface is of the order of $\frac{1}{2}$-minute. Making a generous allowance for friction on the cover and also taking into account the fact that the average diameter of cross-section of the R.100 or R.101 is less than 60ft. it can thus be asserted that the average time during which a particle of rain remains on the cover is not greater than $\frac{1}{4}$-minute. This signifies that the amount of " free " water thereon is not greater than that which would be deposited during precipitation on a surface equal to the horizontally projected area of the ship in a period of $\frac{1}{4}$-minute. For the R.100 or R.101 this area is of the order of 50,000 sq. ft., and if we consider the maximum rainfall, as before, as corresponding to a cloudburst, when the precipitation may be 4in. per hour or 1/60in. per $\frac{1}{4}$-minute, the corresponding weight of " free " water works out at about $2\frac{1}{2}$ tons.

This is almost certainly an over-estimate, and in addition corresponds to the very worst possible atmospheric conditions, so that, taking into account also the small " soakage " of the outer cover (paragraph 11) and the negligible " impact " effect, it may be definitely asserted that there is little chance of a large airship being " beaten down " by a rain storm as such. This, of course, is distinct from the effect of any vertical air currents present with the latter. Hence, the " intermittent rain " which the R.101 experienced during her last flight must have had as negligible a direct effect in bringing about her destruction as the " soakage " of the fabric.

It is of interest to note that since the displacement of an airship varies roughly as the cube of its over-all dimensions, the " soakage " and " impact " effects as their square, and the " free " water effects as their 2.5th power, a large ship will, contrary to a fairly widespread belief, be less inconvenienced during a storm than a small one, owing to the fact that any forces due to the above are a relatively smaller percentage of the displacement. This is still another, although minor, argument in favour of the " big ship."

In conclusion the writer would like to express his appreciation of the assistance rendered by his late colleagues in the Airship Guarantee Co. during the

[26] See also a reference in the paper already quoted by Scott and Richmond.

preceding investigations, especially in the experimental work entailed. He also wishes to thank the above Company and the Air Ministry for permission to publish the results of this research work.

Appendix I.

Useful Data in Airship Design

(*a*) *Conversion of Units*

Length.—1 inch = 25.4 mm.; 1 mm. = .0394in.; 1 metre = 3.281ft.; 1ft. = .305 metre.

Area.—1 sq. metre = 10.764 sq. ft.; 1 sq. in. = 6.452 sq. cm.

Volume.—1 cu. metre = 35.317 cu. ft.; 1 cu. ft. = .283 cu. metre.

Mass.—1 kilogram = 2.205lbs.; 1 millier or metric ton (1,000 kilos) = .9842 tons.

Force.—1 dyne = force necessary to give a mass of 1 gr. an acceleration of 1 cm. per sec. per sec. $= 2.25 \times 10^{-6}$ lbs. mb.

Work.—1 erg. = work done when force of 1 dyne is moved through 1 cm. $= 7.373 \times 10^{-8}$ ft./lbs.

Pressure.—1 millibar (mb.) = 1,000 dynes per cm.^{2} $= 1.45 \times 10^{-2}$ lbs./m.^{2}; 1 mm. of mercury at 0°C ($g = 980.62$ cm./sec.^{2} $= 1.3332$ mb. $= 4/3$ mb. very nearly; 1 inch of mercury at 0°C. ($g = 980.62$ cm./sec.) $= 33.862$ mb.

Temperature.—°A. = °C. + 273 = (°F. + 459.21)/1.8.

(*b*) *Binomial Theorem*

$$(1+x)^n = 1 + nx + [n(n-1)/2.1]\,x^2 + [n(n-1)(n-2)/3.2.1]\,x^3 + \ldots$$

for all real values of x and n, $+ve$, $-ve$, and fractional.

If α, β, $\gamma \ldots$, are small relative to unity

$$(1+\alpha)(1+\beta)(1+\gamma)\ldots = 1 + (\alpha+\beta+\gamma+\ldots) \text{ approx.}$$

$$1/(1+\alpha) = 1-\alpha, \quad 1/(1-\alpha) = 1+\alpha$$

Appendix II.

Purity Meters

Various methods, both physical and chemical in nature, have been proposed, and in some cases developed, for measuring the purity of the lifting gas of airships, but as far as hydrogen is concerned these can for practical work be reduced to two, based upon the following phenomena respectively:—

(*a*) Thermal conduction from a hot wire.

(*b*) Effusion through a fine orifice.

An instrument making use of (*a*) depends in principle upon the fact that the thermal conductivity of hydrogen is very much greater than that of air, so that an electrically heated wire immersed in an atmosphere of the first gas will cool at a very much greater rate than if in one of the second. The temperature of such a wire, and hence its electrical resistance, will therefore vary very widely if exposed to a mixture of hydrogen and air, according to the relative proportions of these gases present, and by a suitable adaptation of the Wheatstone bridge principle or otherwise, an ammeter can be calibrated to indicate "percentage purity" directly.

Although the "hot wire" method has long been used for measuring the thermal conductivity of gases, the first practical application of it to purity measurement was made in 1916 by Dr. G. Shakespear through his invention of the "katharometer," now widely used in airship work.

This instrument, which is very portable and convenient, has, however, been fully described in various technical publications[27] and will not be dealt with further here.

The alternative method (*b*) was originally suggested by Bunsen as one of utility for determining the density of gases, and at the present time there are two well-known instruments used for the purpose and based upon this principle, the "Schilling Diffusion Apparatus" and "Simmance's Specific Gravity Bell." Both depend for their functioning upon the fact that the escape of gas through a hole which is not too fine and long obeys Torricelli's theorem, so that the velocity with which the gas streams through the aperture is proportional to the square root of its pressure divided by its density; hence, for different gases under the same pressure the velocity will vary inversely as the square root of their density.

Mathematically this can be shown to be expressed by:—

Volume of gas discharged in time dt

$$=\delta \cdot a\sqrt{2p/\rho}\, dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (i)$$

where δ is the "coefficient of discharge" for the orifice.

a is the area of the latter.

p is the excess pressure forcing the gas through the orifice.

ρ is the density of the gas.

Diagrammatically the apparatus may be represented as in Fig. 8 and is of the simplest construction. The method of operation is as follows:—A metal or glass "bell," open at one end, is partially immersed in water with this end downwards, and the whole is counter-weighted so that the excess pressure to which the gas in the interior is subjected is very small (about 1/12lb. per sq. in.). At the top is a cock, which on opening allows the imprisoned gas or air to escape through a small orifice; the "bell," therefore, slowly sinks in the water, and the time which is taken to descend through a certain distance, as denoted by two fiducial marks, is noted with a stop-watch. If then the "bell" be filled alternately with atmospheric air and with the gas whose purity is to be tested, the ratio of these times of descent can be easily shown to be a function of the "percentage dry purity" of the gas.

Since the velocity of descent is very small the acceleration of the "bell" is negligible, so that, practically speaking, it is in equilibrium in all positions between the three forces represented by (i) its weight, (ii) the buoyancy of the actual amount of submerged metal or glass, (iii) that force due to the excess pressure of the imprisoned gas acting over the area of the closed end. Of these the second can be easily shown by a numerical calculation to be negligible in comparison with the other two, and hence, since the first is constant in all positions, it follows that for equilibrium the excess pressure of the gas must have a constant value which is also independent of the nature of the latter. From the figure it is then clear that if A be the cross-sectional area of the "bell" at its "water line," and the former descends a small distance dx in the time dt, then substituting in the equation (i)

$$A dx=\delta \cdot a\sqrt{2p/\rho}\, dt$$

If, as in practice, the "bell" is cylindrical, then A is a constant; also p, as we have seen, has a fixed value, and since the absolute pressure $(P+p)$ is also a constant during the operation, so is ρ. Hence, integrating to find the time T which it takes for the bell to descend through a total distance x, we have:

$$Ax=\delta \cdot a\sqrt{2p/\rho}\, T \quad . \quad . \quad . \quad . \quad . \quad (ii)$$

Let now the "times of descent" for the air and impure gas be denoted by T_A and T_G and the densities by ρ_A and ρ_G. Also let δ_A and δ_G be the "coefficients

[27] e.g., Shakespear, R. & M. Nos. 316, 516, 517. Shakespear and Daynes, *ibid*, 1917 No. 367; 1918 No. 435; also Daynes, Roy. Soc. Proc. A., 1920, xcvii, 273.

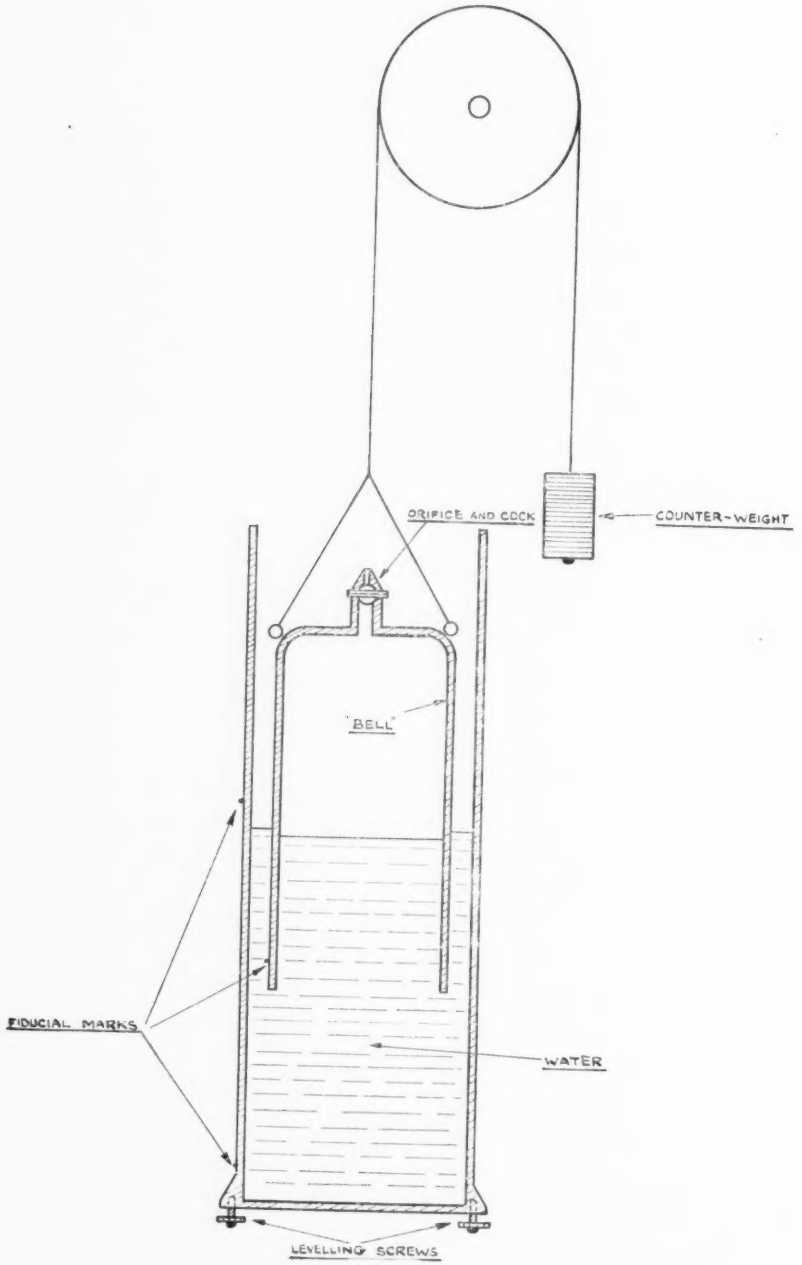

ORIFICE AND COCK
COUNTER-WEIGHT
"BELL"
FIDUCIAL MARKS
WATER
LEVELLING SCREWS

FIG. 8.

of discharge " in the two cases, if these be unequal. Then we have, from equation (ii) :—

$$A\varkappa = \delta_A a \sqrt{2p/\rho_A T_A}$$
$$A = \delta_G a \sqrt{2p/\rho_G T_G}$$
$$\text{by division } 1 = (\delta_A/\delta_G)\sqrt{\rho_G/\rho_A}\, T_A/T_G$$
$$\text{whence } \rho_G/\rho_A = (\delta_G/\delta_A)^2 (T_G/T_A)^2 \quad . \quad . \quad . \quad . \quad \text{(iii)}$$

Both the air and impure gas, when drawn into the " bell," will become saturated by the pressure of the free water surface, so that, neglecting the slight excess pressure inside we shall have from equation (x) of paragraph 2, putting $r = 1$:—

$$\rho_A = (28.971\ P/RT)\,[1 - (10.955/28.971)\,(\pi_T/P)] \quad . \quad . \quad \text{(iv)}$$

where P is the atmospheric pressure, as in paragraph 6.

T is the atmospheric temperature.

π_T is the atmospheric saturated vapour pressure of water at temperature T.

The impure gas, when taken from the container, may be slightly " superheated," and so have a temperature $(T+t)°$A. Since, however, the ratio of hydrogen (or helium) to air molecules is the same in both the container and " bell," so that the " percentage dry purity " K is the same, we have, for the case of hydrogen, from equation (xi) of paragraph 2 :—

$$\rho_G \text{ for hydrogen} = [28.971\ P/R\,(T+t)]\,(1 - 26.955\ K/28.971 + \{\,26.955/28.971\,\}\ \{\,K\pi_{T+t}/P\,\} - \{\,10.955/28.971\,\}\ \{\,\pi_{T+t}/P\,\}) \quad . \quad \text{(v)}$$

Substituting from (iv) and (v) in (iii), cancelling, and transposing :—

$$\frac{\{\,[1 - 26.955\ K/28.971 + (26.955/28.971)\,(K\pi_{T+t}/P) - (10.955/28.971)\,(\pi_{T+t}/P)]\,\}}{[1 - (10.955/28.971)\,(\pi_T/P)]} = (T - t/T)\,(\delta_G/\delta_A)^2\,(T_G/T_A)^2$$

$$\text{whence } 1 - (T - t/T)\,(\delta_G/\delta_A)^2\,(T_G/T_A)^2$$
$$= [(26.955/28.971)\ K - (26.955/28.971)\,(K\pi_{T+t}/P) + (10.955/28.971)\,(\pi_{T+t}/P) - (10.955/28.971)\,(\pi_T/P)]/[1 - (10.955/28.971)\,(\pi_T/P)]$$
$$= (26.955\ K/28.971)\,[1 - \pi_{T+t}/P + (10.955/26.955\ K)\,(\pi_{T+t}/P) - (10.955/26.955\ K)\,(\pi_T/P) + (10.955/28.971)\,(\pi_T/P)]$$

The expression within square brackets can be easily shown, by similar treatment to that in paragraph 6 to reduce for practical purposes to $(1 - .57\ \pi_{T+t}/1{,}000$ mb.). Also, it is found experimentally that δ_A and δ_a do not vary sensibly with the amount of impurities in the hydrogen or moisture in the atmosphere; and hence $(\delta_G/\delta_A)^2$ is a constant $= \omega$, say.

The above thus reduces to

$$K = 1.0744\ [1 - \omega\,(T - t/T)\,(T_G/T_A)^2]\,(1 + .57\ \pi_{T+t}/1{,}000 \text{ mb.})$$

for hydrogen, and similarly

$$K = 1.1602\ [1 - \omega\,(T - t/T)\,(T_G/T_A)^2]\,(1 + .57\ \pi_{T+t}/1{,}000 \text{ mb.})$$

for helium, the value of ω not being necessarily the same in the two cases; from which " percentage dry purity " of hydrogen

$$= [107.44 - 107\ \omega\,(T - t/T)\,(T_A/T_G)^2]\,(1 + .57\ \pi_{T+t}/1{,}000 \text{ mb.}) \quad \text{(vi) } (a)$$

and " percentage dry purity " of helium

$$= [116.02 - 116\ \omega\,(T - t/T)\,(T_A/T_G)^2]\,(1 + .57\ \pi_{T+t}/1{,}000 \text{ mb.}) \quad \text{(vi) } (b)$$

The constant ω in both cases will probably not differ much from unity, and can be determined experimentally by noting the " times of descent " when the " bell " is filled alternately with air and hydrogen (or helium) of known dry purity; the " test " sample, for instance, could be chemically pure gas (*i.e.*, except for the presence of water vapour), prepared, in the case of hydrogen, by electrolysis. The " humidity correction " $(1 + .57\ \pi_{T+t}/1{,}000$ mb.) is of exactly

the same form as the correction for converting "percentage purity," as commonly defined, into "percentage dry purity" (paragraph 6); it can therefore be read directly from the 100 per cent. curve of Graph D, and in practice the percentage value is simply added to that of $[107{\cdot}44 - 107\ \omega\ (T-t/T)\ (T_G/T_A)^2]$ or $[116{\cdot}02 - 116\ \omega\ (T-t/T)\ (T_G/T_A)^2]$, whichever gas is being examined.

Combining the above with equations (xxi) (*a*) and (*b*), in paragraph 6, it will be seen that the "percentage purity" of hydrogen, as commonly defined, is given the expression

$$[107{\cdot}44 - 107\ \omega\ (T-t/T)\ (T_A/T_G)^2]\ (1 + {\cdot}57\ (1-r_G)\ \pi_{T\,t}/1{,}000\ \text{mb.})$$

where 100 r_G is, of course, the relative humidity of the hydrogen when in the container; similarly in the case of helium. Since it is sometimes difficult to find this relative humidity it is evidently preferable from the standpoint of accuracy and facility of measurement, as well as from the mere theoretical considerations dealt with in paragraph 6, that K should be utilised in practical work instead of K'.

The use of the "Simmance's Specific Gravity Bell," or the "Schilling Diffusion Apparatus," is somewhat awkward in practice, as such instruments have to be levelled and can only be used on an airship when the latter is at rest in a shed. In addition, the first at least is somewhat heavy and cumbersome to carry about and in both cases calculations have to be made.

A very portable and convenient little instrument might, however, be developed indicating percentage dry purity directly and without the necessity of applying corrections based upon comparison of the velocity of sound in the impure gas and the atmosphere. This method has been adopted in another form for the detection of "fire-damp" in mines, and is exceedingly promising, since the alteration of this velocity, corresponding to the presence of small quantities of air and water vapour in hydrogen (or helium) is very large.

As shown in any text-book on Sound, the velocity of sound in a free gas is given by

$$U = \sqrt{(\gamma P/\rho)} \qquad \text{(vii)}$$

where P is, as usual, the pressure of the gas, ρ its density, and γ is the ratio of its specific heats at constant pressure to that at constant volume.

The velocity of sound in a pipe differs in a small degree from that given by the above expression,[28] but this discrepancy will be neglected as a first approximation.

Again, the fundamental pitch or frequency of a pipe closed at one end is given approximately by the expression

$$n = U/4l \qquad \text{(viii)}$$

neglecting "end effect," where n is the frequency and l the length of the pipe. If then we arrange two such pipes, such that one contains the gas to be tested from the container, and the other air, then on adjusting the length of one relative to the other such that the fundamental note which they emit when sounded is the same, we shall have

$$n = \sqrt{\gamma_G P/\rho_G}/4l_G = \sqrt{\gamma_A P/\rho_A}/4l_A$$

where the subscript $_G$ refers to the gas, and $_A$ to the air, P being the same for each very nearly. From this we find

$$l_A/l_G = \sqrt{(\gamma_A/\gamma_G)}\ (\rho_G/\rho_A)$$
$$\text{or } \rho_G/\rho_A = (\gamma_G/\gamma_A)\ (l_A/l_G)^2 \qquad \text{(ix)}$$

Now it can be shown that γ_G remains sensibly constant and equal to γ for hydrogen (or helium), for all mixtures found in practical work, and similarly for γ_A. The above equation is then similar in form to equation (iii), so that by substituting for ρ_G and ρ_A we shall deduce expressions analogous to those given

[28] A. B. Wood's "Sound," Section III.

by equations (vi) (*a*) and (*b*) for the diffusion apparatus. In order to avoid the necessity of making any corrections for superheating and humidity, however, it is suggested that both the gas from the container and the air, before being conducted into their respective pipes, should be passed through a simple desiccator and also through their spiral tubes arranged so as to equalise their temperatures by thermal conduction. Evaluating (γ_G/γ_A) for the cases of both hydrogen and helium, neglecting the humidity correction, and putting $t=0$, we thus find that

" Percentage dry purity " for hydrogen

$$=[107.44-107\ (l_A/l_G)^2] \quad . \quad . \quad . \quad \text{(vii) } (a)$$

and " percentage dry purity " for helium

$$=[116.02-134\ (l_A/l_G)^2] \quad . \quad . \quad . \quad \text{(vii) } (b)$$

The two pipes could most conveniently be arranged as in the " Binaural Compensator " used in apparatus for the directional reception of sound;[29] the ratio of their lengths would then depend upon an angular position, so that (since K is a function of l_A/l_G only) a dial could be calibrated to indicate " percentage dry purity " directly and without any necessity for corrections.

The sounding of the pipes could best be effected by diaphragms at the " open " ends, each of these being maintained in oscillation at the frequency, corresponding to the length of the pipe and the nature of the gas it contains, by a small valve oscillator, such as is well-known in acoustic work. The currents in the former would then be led to two telephones, and if the lengths of the tubes were not correctly adjusted, the fact could be fairly accurately detected by the lack of tune in the notes emitted. For a final adjustment the said currents could be " heterodyned," and passed through an A.C. ammeter of quick period; at the critical point corresponding to complete synchronism between the notes any " flicking " of the pointer of this ammeter would of course cease.

It is believed that such an instrument is quite practicable, and would obviously be very rapid in operation, due to the avoidance of any necessity of time calculation, and the substitution of direct readings.

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[29] Cf. A. B. Wood's " Sound," p 413, or " The Mechanical Properties of Fluids " (Blackie & Sons, Ltd.), p. 295.

HIGH-SPEED FLYING

BY

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(The substance of a lecture delivered to the British Association for the Advancement of Science on 29th September, 1931.)

The attainment of the mighty speed of 379* miles an hour by an officer of the Royal Air Force (Flight Lieutenant Stainforth) is fresh in our minds and we honour the pilots, the aircraft builders and the engine constructors who made this possible. We remember also, I hope, the scientific workers, whose investigations afforded the basis on which so much else was built. When one contemplates motion at such speeds as this, one cannot help speculating where such enterprise will lead. The power to move is not new; railways and roadways give abundant evidence of high speed, but motion along them is almost entirely in two dimensions. It is only when the freedom of the air makes three-dimensional motion possible that the problem appears in its greatest and most interesting aspect. In the air there is combined the greatest possible freedom with the highest possible speed, and it is this combination which makes the study of such intense scientific interest. Moreover, the flight path may be greatly curved and that curvature may be in any plane. Great forces may then arise and act upon the aircraft from any side. It is not for nothing that the pilot is strapped into his seat.

Catapults

Let us consider first how such speeds are built up. The most rapid way in which an aircraft can be put into a condition of flight is by the use of one of the great catapults built for the purpose of getting aircraft up to flying speed from aerodromes of very restricted area. The most restricted of all such areas is the deck of a ship. These devices can be operated either by compressed air or by the burning of cordite. The expanding gases act inside the telescopic cylinders and give rise to a very powerful thrust. This thrust amounts to many tons, no less in fact than some three times the weight of the aircraft. Now, if the force were but exactly equal to the weight of the aircraft, the rate at which speed would be gained would be exactly equal to that achieved by any freely falling body, namely, 32 feet per second every second. This rate of getting up speed we call 1g. If the force were increased to three times the weight of the aeroplane the acceleration would be 3g, and such in point of fact is about the force actually employed in these catapults. Such an acceleration would give an aeroplane a speed of 70 miles per hour at the end of 60 feet travel along the catapult. To picture the rapidity of this gain of speed, bear in mind that the normal rate of gain of speed by a railway train is but one hundredth part of this. That is a steam train. An electrical one has an acceleration of three times as much, but still vastly below that of the aeroplane catapult. A motor car might get to within one-sixth of it, if not further. The achievement of speed by an ordinary railway train is surprisingly slow, for although it may be capable of travelling at 80

* Since increased to 407.5 m.p.h.

miles an hour, it cannot actually cover as much as a yard in the first second from its start. Animals, even gorillas, make a slow start in comparison with the catapulted aeroplane, though rapid compared with the train. Professor A. V. Hill has pointed out that similar animals built of similar materials should, whatever their size, have the same maximum linear velocity motion. The racehorse and the greyhound have much the same speed. In the case of a runner on the track the limit to the rate of picking up speed is given by the adhesion between the leather and studs on his shoes and the ground. Strength greater than this would cause slipping to occur and might actually be a hindrance rather than a help. The catapulted aeroplane, rapidly as it does start, is far from holding the record for high acceleration. The piston in a motor car engine begins each stroke with an acceleration 100 times as fast, whilst that of a rifle bullet is a hundred times as rapid as even that. Hence the bullet starts 10,000 times as rapidly as the aeroplane and a million times as fast as the steam train.

I think that the highest acceleration ever attained on this planet by a man-built mechanism must be that produced in the air-driven centrifuge used by Dr. Beams and Mr. Weed, of the University of Virginia. My attention was drawn to this by Dr. Bourdillon, of the Medical Research Institute. This apparatus is stated to have achieved the amazing rotational speed of no less than 500,000 revolutions per minute, giving a centrifugal acceleration of one million times that of gravity.

The apparent weight of an ounce of metal under these conditions would rise to 30 tons. In this little apparatus there has, therefore, been produced on the earth an acceleration actually exceeding that at which on the dark companion of Sirius, a body falls to the surface. An astronomical friend tells me that it is probably higher even than that on Van Maanen's star, the densest star known in the entire Universe. Perhaps the first "record" for the Universe!

Let us now think of the aeroplane pilot. He is sitting in a catapulted aeroplane and suddenly finds a force acting on his body equal to three times his weight. This force acts in a horizontal direction and on every part of him. He can endure it without harm provided he rests his head against a strong rear support. Without some such preparation he would suffer. Sudden changes of speed are bad for the neck, in fact, we usually reserve the severest of them for our worst criminals. If we tried to start our pilot, however, with the acceleration of a motor car piston, his effective weight would become 20 tons and his shape, alas, that of the pancake.

Aerobatics

So far we have only considered motion in a straight line. The really interesting things begin to happen when one is flying in a curved path, doing twists and turns, in fact carrying out what are known as aerobatics. The pull of the earth on the moon does not cause the moon to come any nearer to the earth, but merely balances the centrifugal force on the moon due to its following a circular path round the earth. In the motion of the earth and the sun it is just the same.

In a rotating flywheel the centrifugal forces are balanced by the tensions in the rim, but if the speed is raised too high the wheel will fly to pieces. The faster a body moves and the more curved is its path, the greater is the amount of the centrifugal force. So, too, in the case of an aeroplane, when it is flying in a circular path or looping the loop; this centrifugal force is balanced by the increased lift on the aeroplane wings, whilst the centrifugal force on the pilot's body is balanced by the greater reaction exerted by the floor or his seat. His body appears to him to weigh much more than usual and he may find it almost, if not quite impossible, to lift his arms or to stand up. What is still more embarrassing is that owing to his blood weighing many times its normal weight, it may gradually leave his brain in favour of a lower and more distensible part of the body. There is a record of as much as 11ozs. having been suffused by an

American pilot. Some aerobatic pilots, it is true, try to avoid the inconveniences that arise from this by wearing tight body belts. The immediate effect of the loss of blood in the brain, though fortunately a temporary one, is the loss of sight. This is so common a feature of the fast flying of to-day, that it is familiarly known as "blacking out." I am no physiologist, so I had better be careful what I say on this subject, but I can at any rate state that it is a commonly claimed experience on the part of aircraft pilots that this "blacking out" is not accompanied by any lack of mental clarity and that control, a blind control, of course, can still be maintained on the flying of the aircraft. It is fairly sure, however, that if the aerobatic performance were to maintain this condition for any length of time, say, by flying fast in tight circles, loss of consciousness would result, though I cannot recollect ever having heard of such a case.

This phenomenon has not a physiological interest only, since if a mode of flying is adopted, even temporarily, which causes the pilot's body to weigh five times as much as usual, the load on the wings of the aeroplane will also be five times the usual amount. Some aircraft are sufficiently stoutly designed to stand a treatment as severe as this and some are not. The stress which comes on the wings of an aeroplane depends not on the static load applied by its weight, but upon the manœuvres to which it is subjected. It follows, therefore, that the designer of an aircraft needs to know how high a load is likely to be imposed by such manœuvres. It is no good asking a pilot for quantitative figures, and it is, therefore, necessary to fit the aircraft with a recording instrument known as an accelerometer, either one which draws a graph of the accelerating force against time or one which records solely the maximum load which has come on to the machine during any particular manœuvre. A figure of 3 or 4g is common in fast turns and in loops. A negative acceleration of 2 or 3g arises from an outside loop or bunt, and in that case a pilot is liable not to "blacking out," but a sort of "redding out." With the information thus obtained one may know that the severest manœuvre desired to be carried out on a particular form of aircraft will bring into being an acceleration not exceeding 5g, and that as the wings have a load factor, as it is termed, of 8, and since 8 is greater than 5, there is just that substantial margin of strength which is necessary. But how is the pilot of this aircraft to know how to confine his aerobatics within the acceleration of 5g? The usual answer is that his training is so long and so careful that he can judge of the force by the feeling on his body and realise quickly when he is getting too close to the danger line. An alternative that has been tried is to fit a measuring mechanism inside the pilot's control lever, which gives the pilot a tactile signal which he cannot mistake whenever the acceleration rises to the safe limit. For the rapid training of pilots such a device would no doubt have its use.

Racing Aircraft

The racing aircraft of the Schneider Trophy type are always built as seaplanes and not as landplanes. This is because to reach such enormous racing speeds, they have to take off and land at speeds of over 100 miles an hour. This would normally be so dangerous on land, or alternatively require such huge aerodromes, that it is best to use a great sheet of protected water, such as the Solent.

With these exceedingly fast machines the greatest care has to be exercised during the actual achievement of flying speed. If the engine is opened up too rapidly, the reaction from the airscrew will tend to turn the machine over sideways, so that one float is submerged more than the other and the seaplane is liable to yaw suddenly. This is dangerous. And so the engine must be opened up gently. Furthermore, when the seaplane attains to flying speed and leaves the water, it does so at a steeper angle than it can steadily maintain, so that it is necessary for the pilot to ease his control lever gently forward. This

manœuvre needs very close judgment indeed, since there is always the risk of overdoing it and so touching the sea again in a kind of ricochet—a phenomenon known in the aeronautical world as " porpoising." If the ricochets are repeated the result is likely to be disastrous.

Moreover, although it is quite impossible to avoid water being thrown up by the floats when taking-off, it is most important that it should not be thrown into the swiftly rotating airscrew or it may damage the machine seriously. Once the machine is in the air, anxieties are lessened. The chief remaining one is whether the cooling arrangements will suffice to keep the engine from becoming overheated. The very heating effect of the speed on the air itself is an obstacle. Dr. A. A. Griffiths has predicted that the temperature taken up by a body immersed in a moving fluid must be affected by the velocity of flow to such an extent that the temperature of the cockpit of an aircraft flying at 360 m.p.h. will exceed the atmospheric temperature by nearly 13 degrees Centigrade. This is important not only as affects engine cooling, but as affects the comfort of the pilot, especially as the temperature rise may be expected to be proportional to the square of the speed. Quick motion through the air has usually a cooling effect, but if it becomes very fast this effect is reversed.

The mere measurement of the speeds attained by these amazing speedy craft is far from simple. The ordinary stop-watch of the race course is quite useless. For measuring the speeds of racing seaplanes over the three kilometre course at Calshot, two cinema cameras are used which, in addition to recording the passage of the aircraft at either end of the course by a series of photographs, record the time at which each exposure is made in terms of vibrations of a tuning fork in electrical connection with counters in both cameras. The accuracy with which the time of any one transit of the aircraft can be determined is about 1/20th second, so that a speed of 350 m.p.h. over three kilometres may be measured to the nearest half m.p.h.

It is extraordinary to think that a winning speed of 45 miles an hour sufficed for the first Schneider Trophy contest of only 18 years ago. Though it is fair to say that had M. Prevost, on his 160 h.p. Gnome-engined Deperdussin aircraft, not at first forgotten to cross the finishing line at the end of the race, his speed would have been about 60 m.p.h. instead of 45. The change to the 340 miles an hour to-day (Flt. Lt. Boothman) is due to two chief factors : the cleaner design of the aircraft and the better engine. For the former we have to thank the wind-tunnel experts and aircraft designers, particularly Mr. Mitchell, of the Supermarine Co., and for the latter the great skill of the aero-engine builders, and especially of that great engineer, Mr. Royce, of the Rolls Royce organisation. His wonderful engine gave over 2,000 h.p., with an amazingly low weight to horse-power ratio. I cannot mention the Rolls Royce firm without remembering that it was just three years before the beginning of the Schneider Trophy contest that the death of Mr. C. S. Rolls took place. His death was a heavy blow to the young science of aviation. He combined a wonderful quality of enthusiasm, immense energy, and great ability. I well remember his describing to me his earliest flights as " like using skates which are ready to slide in every direction at once."

Future High-Speed Flying

I come now to the future. Where is one to look for a limit to the speeds which are possible? How fast can man go?

To carry the load of the aircraft is simple. One merely sets the wings to the particular angle of attack which will give the lift one wants. The difficulty is increasing drag. Although I do not intend to trouble you with aerodynamic formulæ, perhaps I should say that at any given height the speed of flight is proportional to the square root of a certain ratio. That ratio is the horse-power

provided per square foot of wing area to the overall drag coefficient of the airplane. For high-speed, therefore, we need a powerful engine, small wing area and a streamlined design. As a rule parts in tandem have less total resistance than the sum of their resistances measured separately, while parts side by side have a greater resistance. This rule needs to be borne in mind when cleaning up a design. Another general rule is that any irregularity near the leading edge of a wing disturbs the boundary layer, reduces what is called the "circulation" and therefore the lift, and increases the head resistance out of all proportion to the size of the obstruction. The flow on the upper surface is far more sensitive in this way than the flow on the under surface. Two years ago Professor B. M. Jones, of Cambridge, lectured to the Royal Aeronautical Society on "The Streamline Aeroplane." His purpose was to show how greatly aeroplane performance could be improved if the air resistance could be reduced to a minimum.

He pointed out that large commercial aeroplanes would, "were they ideally streamlined, either fly at the present top speed for one-third the present power, or, alternatively, travel some sixty miles per hour faster for the same power. Two-thirds of the power used by these aeroplanes is being expended on the generation of turbulence which, from the aerodynamic viewpoint, is unnecessary."

He added that although "It is not suggested that it is easy to design a streamline aeroplane which will be also a practical machine, the immense saving in power, and therefore fuel consumption, which would apparently follow such a step, forces me to the belief that design will evolve steadily in this direction and that the ultimate aeroplane will be as well streamlined on the whole of its external surface as, say, the bottom of a racing yacht or the externals of an albatross. I am fortified in this belief by surveying the animal kingdom. Those birds and fishes which depend on speed for their existence have long since solved the problem. The compromise with structural difficulty was no doubt as difficult for them as for us, but it has ended in the complete triumph of the external form. We can only hope that it will not take us so long to reach this point as, if we are to believe the comparative anatomists, it took them."

To fly near the ground at speeds much above the present record of 379 miles an hour must be almost impossible without a drastic change in aeroplane construction. The only alternative I see to such a change would be the acceptance of a landing speed so high as to be dangerous to life. The relationship of air-speed to horse-power available and horse-power required on a typical aeroplane is commonly such as to give a ratio of the highest speed to the lowest approaching three to one. Suppose we could get a ratio of three and a half to one. Then if the limit to safe landing speed were 110 m.p.h., a top speed exceeding 385 m.p.h. could only be obtained by the introduction of some such device as changing the wing area whilst in flight, or using flaps or slots to reduce the landing speed.

I here refer only to horizontal speeds. By diving vertically even with the aircraft normally used by the R.A.F., speeds as great as the greatest I have named can be, and have been, attained. Such manœuvres are called terminal velocity nose dives—Schneider Trophy aircraft are not put into such dives, but if they were the terminal velocity would, I suppose, approach the enormous speed of 650 miles an hour, which is just about the velocity of sound. Hence any individual standing on the ground below the point in the sky where this diving manœuvre was being carried out, would not hear the machine coming on him, and the first, and last, sensation he received might be the impact of its propeller!

The drag of any body moving through the air increases with the square of the speed and goes on doing so until the velocity of sound, some 700 miles an hour, is reached. Then there is a great change, the drag suddenly more than doubles. Once this barrier is passed the resistance law becomes again proportional to the square of the speed, but with a higher multiplying coefficient. So I should not care to predict that passage beyond that barrier will ever be possible to aircraft as we know them. A rifle bullet does so, of course, as it moves with more

than twice the velocity of sound, but I do not see manned aircraft flying at such speeds.

Without going into such " soundless regions " of speed as I have rejected as unsuitable, if one nevertheless seeks speeds very much higher than those now usual, one can find them by deserting the levels in which Schneider Trophy winners fly and ascend to altitudes of twenty, fifty, and even one hundred thousand feet. There one would find so greatly reduced an air density (about one ninth at 50,000 feet, for example), that the resistance to motion would be correspondingly reduced. A practical, but disturbing example of the greatly reduced resistance to motion in the upper regions of the atmosphere is seen in the performance of the long-range gun which shelled Paris during the War from a distance of no less than 70 miles. Its tremendous muzzle velocity of one mile per second enabled the projectile quickly to leave behind the denser parts of the atmosphere, and reaching a height of no less than 24 miles to travel most of its journey in a region where the pressure was scarcely more than that in a vacuum.

Unfortunately for our flying machine, the air supply for the engine would be drawn from an equally attenuated atmosphere and compressors known as super-chargers would be needed. They take power to drive and an allowance must be made accordingly. Then the airscrew has very tenuous air to bite on and with a limit set by engine stresses to the revolution rate a very high pitch is called for; a pitch possibly inconveniently high for taking-off. As, however, the engine only uses its fuel to turn the airscrew and as the sole object of the airscrew is to drive air behind it in the form of a jet, why not, it may well be asked, burn the fuel so as to cause an air jet at once? This brings us to the rocket-propelled aeroplane, and what may come from that it is early to say. I have seen it claimed from a Heylandt rocket that with a weight of slightly over 8lbs., a propulsive force of not less than 264lbs. is attainable. This is stated to be fed on liquid oxygen and gasolene. By resort to jet propulsion it is not easy to see any theoretical limit to the altitude attainable and scarcely any limit to speed, until one reaches the point at which the intensity of the air friction causes the aircraft to get exceedingly hot, leading in the end to those extreme temperature conditions, and most dazzling paths, of the meteors of the night sky!

In all this one must not forget the passengers who will no more be contented with such tenuous air than is the engine. It is true that the supercharger which feeds the engine can also feed them, but such air would prove an oily, unattractive mixture for breathing. I rather see the passengers by this highly elevated airway seated in an enclosed cabin and fed with oxygen; the cabin having the same kind of absorbents for carbon dioxide and water vapour as are commonly employed in submarines. To some, however, I fear that confinement in a hermetically-sealed cabin may make such a mode of travelling more suitable in their view to the next generation than their own, but it is, after all, only the next generation that is likely to have the chance.

How is one to sum it all up? Here is a tremendous effort to attain speed, and speed alone. But who wants speed divorced from almost every other quality? Certainly not the Royal Air Force and still less the public—so long, at any rate, as speed and noise are such inseparable companions. Presumably the founder of the Schneider Trophy race thought chiefly of speed, but the quest of the Air Ministry in taking part in this endeavour has not been speed as such, but the acquirement of knowledge of how best to reduce air resistance and to increase engine power. The former knowledge once it is attained enables Service and Civil aircraft of the highest aerodynamic efficiency to be designed, and the latter to make possible the construction of reliable engines giving a maximum of power with a minimum of weight. The present remarkable position of British aeroplane design is evidence not only of the rightness of having taken part in this enterprise, but of the success of British aircraft and engine constructors in making full use of it. The knowledge so won filled a gap which badly needed filling; the other

problems of the air which now claim our attention will most fittingly be dealt with in other ways.

The lecture was illustrated by lantern slides, and at its end two cinema films were shown. One of the films—made by the R.A.E.—showed a Virginia being catapulted by the rope and winch method, the winch being worked by compressed air; this was followed by views of a Hawker Hart and a Fairey IIIF machine being catapulted by cordite charges. These views were followed by a series of two "take-offs" and "landings" by one of the Schneider Trophy aircraft.

The second film, which was kindly furnished by the Paramount Company, showed Flight Lieutenant Boothman in a S.VI.B. aircraft winning the Schneider Trophy, and Flight Lieutenant Stainforth beating the World's record on the same machine.

THE TURBO-COMPRESSOR AND THE SUPERCHARGING OF AERO ENGINES*

BY

F/O. F. WHITTLE, *Associate*

Introduction

If the amount of printed matter published on a technical subject is any guide to the general state of knowledge on that subject, then neither the turbo-compressor nor the general problem of the supercharging of aircraft engines can be very well understood at present, much less the combination of the two. Apart from the brief treatment accorded to the behaviour of engines at high altitudes, and supercharging in the well-known text-books on engines, the only printed matter available on the subject is contained in papers published in technical journals (so far as one knows), viz. :—

" Supercharged Aero Engines," R. F. R. Pierce, the JOURNAL of October, 1926.

" The Supercharging of Aircraft and Motor Vehicle Engines," A. H. R. Fedden, the JOURNAL, October 1927.

" Superchargers and Supercharging," Oscar W. Schey, *Aircraft Engineering*, June, 1931.

There is one book in the English language on turbo-compressors, viz. :—" Turbo-Blowers and Compressors," W. J. Kearton.

For a general survey of supercharging methods, etc., the reader is particularly referred to the paper by Mr. A. H. R. Fedden, and to the book by Kearton for more detail of the theory and operation of turbo-compressors than will be given here.

The importance of supercharging cannot be over emphasised, and yet at present a supercharger appears to be regarded as only a useful auxiliary instead of an essential. However, there is a growing realisation of the possibilities of high altitude for economical flying (as witness the fact that the Junkers and Farman Companies—two of the best known continental aircraft manufacturers—are building aircraft for high altitude flying) and when this frame of mind becomes more general the importance of the supercharger will become manifest.

The object of this paper is to try to develop in fair detail some of the theory and problems peculiar to supercharging by means of the turbo-compressor. It is not intended here to discuss the requirements of the airscrew, etc. The whole subject will be treated only as it concerns the engine.

The general theory of the turbo-compressor will be given as briefly as possible, chiefly for the benefit of students, since, as above mentioned, there is very little in print on the subject.

The writer is grateful to the B.T.H. Co., Ltd., for permission to publish test results which they have kindly supplied. As these results were received after the completion of this paper, they have been incorporated in an appendix.

* Any opinions or original theory contained in the following are the Author's own and carry no official sanction.

The Turbo-Compressor

General Principle.—The turbo-compressor is a special type of centrifugal compressor, *i.e.*, one in which means are provided for converting the Kinetic energy with which the fluid leaves the impeller into pressure energy. Where no means for doing this are provided, the pressure head is the centrifugal head, and the leaving velocity is wasted.

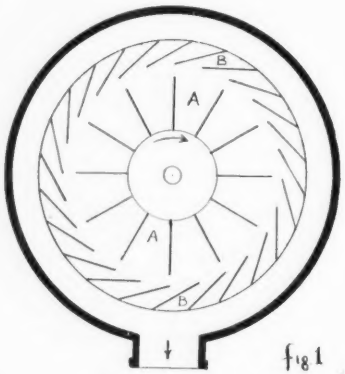


Fig 1

Fig. 1 is a diagrammatic representation of a turbo-compressor. Air enters the blades of the impeller A and being whirled round, is compressed by the action of centrifugal force. On leaving the impeller blades it has a high tangential velocity which is converted into pressure by the fixed diffuser blades B which are designed to perform this function. The device is extremely simple mechanically, as there are no reciprocating parts and no valves. However, very high speeds are necessary to produce an appreciable compression. The most striking feature is the enormous capacity of which such a compressor is capable in proportion to its size.

Types of Rotor.—Turbo-compressors may vary very much in detail—they vary in the rotor construction, the diffusers and the inlet.

The three main types of rotor are shown in Figs. 1, 2a, and b; Fig. 1 shows the straight radial vaned type. This type is the most suitable structurally for high speeds. In this type the increase of pressure energy should be roughly equally shared by the impeller and the diffusers.

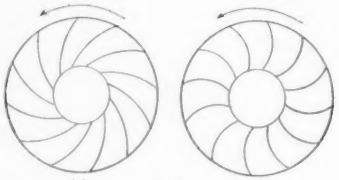


Fig 2

Fig. 2a shows a type in which the blades curve backwards; in this case the rise of pressure energy is greater in the impeller and less in the diffusers than with the type shown in Fig. 1. Fig. 2b is another curved vane type, but a type in which the greater portion of the pressure rise takes place in the diffusers. The straight vaned type is the most common form used in supercharging, and will be the type referred to throughout this paper.

Rotors vary also in construction. They may be doubly shrouded (*i.e.*, the vanes are mounted between two discs), singly shrouded, or unshrouded. The unshrouded type is the strongest in rotation if vibration of the tips can be prevented, the other types being more limited by the stress in the shrouds.

Types of Diffusers.—There are two general types of diffuser arrangement :—

(*a*) In which the fluid is allowed to form a free spiral vortex.

(*b*) In which blades are placed to form a series of divergent passages (as in Fig. 1).

The two types may overlap largely, *e.g.*, the inner part of the Diffuser annulus may be a " vortex space " and the outer part may be fitted with blades. The bladed type permits of a smaller diameter than the " vortex " type, but the latter has other important advantages which will be discussed later.

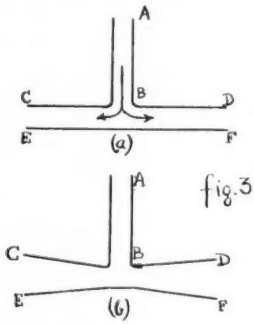


fig. 3.

Theory of the Turbo-Compressor

Radiating Current.—If a fluid travelling in a pipe AB (Fig. 3*a*) be constrained to flow outwards between two parallel surfaces CD and EF, distance a feet apart, then since at increasing radius the cross section of the flow increases, the velocity must decrease. If the fluid is incompressible, then the velocity v and radius r at any point are such that $2\pi rav$ = volume flowing per second = a constant, *i.e.*, $rv = k$. The pressure and velocity at any point are such that $P + \rho v^2/2 = k'$ (Bernouilli's equation), where P is the pressure at that point and ρ is the density in slugs, k' is a constant. Substituting k/r for v then $P + k''/r^2 = k'$, *i.e.*, as r increases the pressure rises. If the surface between which the radiating current flows diverge, as in Fig. 3*b*, then the velocity is also inversely proportional to the distance between the surfaces.

Free Circular Vortex.—A free circular vortex in a fluid is a state of circular motion where the motion at any time is governed entirely by the forces due to the velocity and pressure distribution in the fluid. A free vortex is a common sight in water (little whirlpools are free vortices) and observers will have noted that the outside portion rotates slowly while the inner portions are whirling rapidly, the pressure in the centre is obviously very much lower than that at the outside.

Referring to Fig. 4, which represents a portion of a free vortex, consider the equilibrium of a small portion of fluid ABCD, of radial width δr and rotating at a radius r with linear velocity v ft. per second. Let the circumferential length of the element, also the depth be unity. The mass of the element is thus $\rho . \delta r$ and its centrifugal force $= \rho v^2 \delta r/r$. If we call the pressure $P + \delta P$ and the velocity $v + \delta v$ at the radius $r + \delta r$ then $\delta P = \rho v^2 \delta r/r$.

From the Bernouilli equation

$$P+\rho v^2/2=(P+\delta P)+(\rho/2)\,(v+\delta v)^2$$

substituting $\rho v^2\delta r/r$ for δP, we have

$$P+\rho v^2/2=P+\rho v^2\delta r/r+(\rho/2)\,(v^2+2v\,.\,\delta v+\delta v^2)$$

from this we obtain $v\,.\,dr+r\,.\,dv=0$, *i.e.*, $rv=$a constant. Therefore the pressure and radius are connected in the same way as for the radiating current, since v is inversely proportional to r. It should be noted, however, that v the tangential velocity is unaffected by the distance apart of the surfaces, in which it differs from the radiating current.

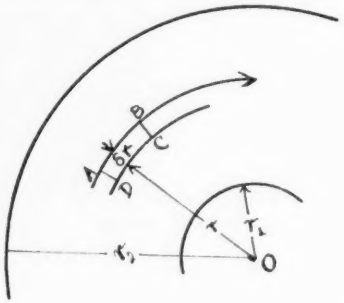


fig. 4.

Free Spiral Vortex.—If a free circular vortex be combined with a radiating current, a free spiral vortex is formed. Referring to Fig. 5, if v_t be the tangential component of the velocity, and $v_r=$the radial component, then the angle $\theta=\tan^{-1} v_r/v_t$. If the surfaces between which the flow is taking place are parallel then $v_r \propto 1/r$ and $v_t \propto 1/r$ $\therefore$ $\tan\theta=$a constant, *i.e.*, $\theta=$a constant, and the

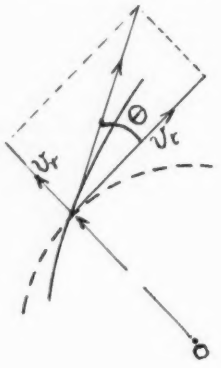


fig. 5.

motion of the fluid conforms to a logarithmic spiral. If the surfaces are not parallel then the radial component v_r also varies inversely as the distance between the surfaces, and v_t is unaffected. The resultant velocity $\sqrt{(v_t^2+v_r^2)}$ is seen to

be slightly affected by the distance between the surfaces, but since as in all cases where free spiral vortices occur in the turbo-compressor v_t is usually three or four times as great as v_r, the variation in resultant velocity due to divergence of the "walls" is negligible, though the effect on the direction of the flow is appreciable. It is approximately true to say in all cases that the resultant velocity is inversely proportional to the radius, and this is exactly true when the walls are parallel. Thus, the greater the radius, the greater the proportion of pressure energy, the total energy being constant at all radii.

In cases where the properties of the free spiral vortex are used to realise pressure from velocity, it is seen that the radial dimension of the vortex space will have to be large to get the best results. This is a drawback to this method of "energy conversion." When the compressibility of the fluid is important, then the flow will be modified—for example, the radial velocity is inversely proportional to the density also, but whether the fluid is compressible or otherwise, the kinetic energy lost in passing from the inner to the outer edge of the vortex equals the pressure energy gained according to the Bernouilli equations

$$P_2 - P_1 = (\rho/2)\,(v_1^2 - v_2^2) \quad . \quad . \quad . \quad . \quad (1)$$

for incompressible fluids and

$$\{\gamma/(\gamma-1)\}\,(P_2^{1-1/\gamma} - P_1^{1-1/\gamma})\,gK^{1/\gamma} = (v_1^2 - v_2^2)/2 \quad . \quad . \quad (2)$$

for compressible fluids (K and γ from the adiabatic law $PV^\gamma = K$). (An ideal process is assumed.)

A Forced Circular Vortex.—A forced vortex is a circular motion in a fluid where the motion conforms to an external agency; in the case we will consider the motion of the fluid is that produced in a compressor impeller by the impeller blades, and the angular velocity of all parts of the fluid is the same and equal to the angular velocity of the impeller (or so we shall assume for the present). Using Fig. 4 again, then the centrifugal force of the small element ABCD $= \rho \,.\, \delta r \,.\, rw^2/g$, where ρ is in lbs. per cubic foot and w = angular velocity, and therefore

$$\delta P = (w^2/g)\,\rho \,.\, r \,.\, \delta r\text{: from } P/\rho^\gamma = K,\ \rho = (P/K)^{1/\gamma}$$

$$\text{and } \therefore \delta P = (w^2/gK^{1/\gamma})\,P^{1/\gamma} \,.\, r \,.\, \delta r \ \therefore \delta P/P^{1/\gamma} = (w^2/gK^{1/\gamma})\,r \,.\, \delta r$$

$$i.e.,\ \{\gamma/(\gamma-1)\}\,(P_2^{1-1/\gamma} - P_1^{1-1/\gamma})\ K^{1/\gamma} = (r_2^2w^2 - r_1^2w^2)/2g$$

If therefore P_1 and r_1 are the pressure and radius at the inner edge of the vortex, *i.e.*, at the impeller inlet, and P_2 and r_2 the pressure and radius at the outer edge (the impeller periphery) then r_2w = peripheral velocity = v_p of impeller and r_1w = velocity of blades at inlet edge = v_1, therefore

$$(v_p^2 - v_1^2)/2g = \{\gamma/(\gamma-1)\}\,(P_2^{1-1/\gamma} - P_1^{1-1/\gamma})\ K^{1/\gamma} \quad . \quad . \quad (3)$$

which is the Bernouilli equation again.

On this reasoning we see that pressure energy due to centrifugal force in the rotor $= (v_p^2 - v_1^2)/2g$ and pressure energy due to diffuser action $= (v_p^2 - v_o^2)/2g$ where v_o is the velocity still remaining at the delivery edge of the diffuser, thus the total energy imparted in the passage through the compressor $= (2v_p^2 - v_1^2 - v_o^2)/2g = v_p^2/g$ if the whole of the inlet were at the exact centre (impossible in practice, of course) and v_o were extremely small. The expression v_p^2/g represents the maximum attainable "work capacity" of a turbo-compressor per pound of fluid (single stage). It is impossible to attain this in practice, but the expression v_p^2/g will be used as a standard for comparison. It is important that the work capacity of an actual compressor should be as near to this as possible in order to keep down the peripheral speed for a given job of work.

The Relative Eddy.—Up to this point it has been assumed that the fluid entering the compressor, immediately takes up the angular velocity of the impeller. This cannot happen in practice, owing to the inertia of the fluid. In fact if it did happen, then the flow through the impeller would be purely radial

relative to the impeller, and therefore there would be no torque reaction on the impeller. If a cylindrical vessel containing fluid were divided into compartments as in Fig. 6a, and rotated with an angular velocity w, then the fluid in each compartment would rotate in the opposite direction relative to the vessel, each particle of fluid completing one revolution relative to the vessel as the vessel itself completes one revolution; this effect is called the "Relative Eddy," by Dr. A. Stodola (Steam and Gas Turbines, Vol. 2). If the fluid were inviscid, then this relative eddy would persist indefinitely. If now a radial flow were imposed on the relative eddy, the resultant flow in each sector would be somewhat as in Figs. 6b and c. In Fig. 6b the radial flow is assumed to be very small and in 6c the

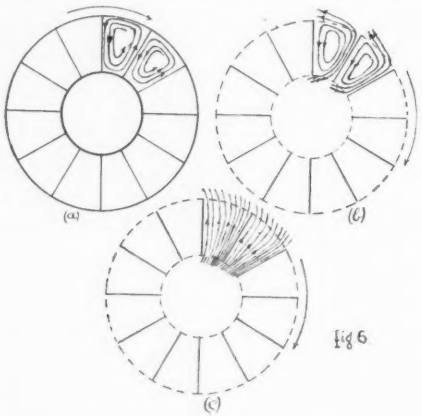


Fig 6

radial flow is large. Any person who has any doubt of the existence of the relative eddy should take a jug of liquid (with particles floating to make the motion visible) and turn round while holding the jug at arm's length. It will be found that until the effect of viscosity becomes appreciable, the liquid will complete one revolution in the jug for every revolution of the jug, but in the opposite sense, *i.e.*, each portion of the fluid maintains its orientation until constrained by viscosity to do otherwise.

In the impeller of a turbo-compressor, the fluid passes through in so short a time that the effect of viscosity on the relative eddy is negligible.

Stodola appears to think that there is also a circulation round the blades analogous to the circulation which is said to exist round an aerofoil. However, this should be accepted with reserve as in aerofoil theory the circulation is necessary to account for the resultant force, whereas the resultant force is explained by the existence of the relative eddy in the case of the compressor impeller.

What are the effects of the relative eddy on the performance of the compressor? Firstly, it is obvious that the tangential component of the velocity with which the fluid leaves the impeller will be less than the peripheral velocity of the impeller by the tangential component of the relative eddy at the impeller periphery. According to Stodola that tangential component of the relative eddy v_e is given by $v_e = (t \sin \beta w)/2$ where t is the pitch of the blades at the periphery, β is the angle of the blades at the periphery and w is the angular velocity of the rotor, $t = 2\pi r/n$ where r is the radius of the periphery and n the number of blades, therefore $v_e = (2\pi r \sin \beta w)/2n$. With straight radial blades $\beta = 90°$ and $v_e = (\pi/n)\, v_p$, where v_p is the peripheral speed of the impeller. The tangential component of velocity of the fluid leaving the impeller is thus

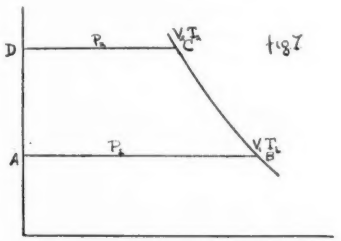
$$v_t = v_p\,(1 - \pi/n) \qquad . \quad . \quad . \quad . \quad . \quad (4)$$

Stodola states: "This does not mean a real energy loss, but only a smaller work capacity of the wheel than if there were no relative eddying."

It is the work capacity of the diffuser space which is affected.

The relative eddy cannot modify the centrifugal head, because it is obvious that the centrifugal force of the liquid in the compartments (Fig. 6a) would be unchanged if the liquid suddenly became solid and the relative eddy impossible; in other words, the pressure head due to the centrifugal force will not be affected.

There is probably a loss due to variation in leaving velocity round the impeller periphery.



Thermodynamics of the Compressor.—The operation of the turbo-compressor may be represented on P.V. and T.ϕ. diagrams. In Fig. 7, a P−V diagram, the difference of pressure energy between P_1 and P_2 is represented by the area $ABCD$, where BC represents the adiabatic compression from P_1 to P_2. $ABCD=\int_{P_1}^{P_2} VdP$. If W ft. lbs./lb. is the work capacity of a compressor then $W=\int_{P_1}^{P_2} VdP=v_p^2/g$ for a compressor where the work capacity is a maximum.

$$W=\int_{P_1}^{P_2} VdP=\{\gamma/(\gamma-1)\}(P_2^{1-1/\gamma}-P_1^{1-1/\gamma})K^{1/\gamma} \quad . \quad . \quad (5)$$

$$=\{\gamma/(\gamma-1)\}(P_2V_2-P_1V_1) \quad . \quad . \quad . \quad . \quad (6)$$

$$=\{\gamma/(\gamma-1)\}R(T_2-T_1)=K_p(T_2-T_1) \quad . \quad . \quad . \quad (7)$$

where P_1, V_1, T_1, and P_2, V_2, T_2, are the pressures, specific volumes and temperatures at the points B and C respectively, R is the gas constant, and K_p is the specific heat of air at constant pressure in mechanical units. From (7)

$$T_2-T_1=\{(\gamma-1)/R\gamma\}W \text{ or } T_2=T_1+\{(\gamma-1)/\gamma\}(W/R)=T_1+W/K_p \quad (8)$$

i.e., the temperature rise is directly proportional to the work capacity at any peripheral speed, and also the end temperature is a linear function of the initial temperature at a given peripheral speed. From (8)

$$T_2/T_1=1+W/(K_pT_1)=1+0.00298\,(W/T_1) \quad . \quad . \quad . \quad (9)$$

($R=96$, $\gamma=1.4$). Now

$$\rho_2/\rho_1=(T_2/T_1)^{1/(\gamma-1)}=(1+0.00298\ W/T_1)^{1/(\gamma-1)} \quad . \quad . \quad (10)$$

and

$$P_2/P_1=(T_2/T_1)^{\gamma/(\gamma-1)}=(1+0.00298\ W/T_1)^{\gamma/(\gamma-1)} \quad . \quad . \quad (11)$$

In practice, owing to various losses (which will be briefly discussed), the work put into the wheel will be greater than the work represented by $ABCD$, and the ratio:—

$$\frac{\text{Work theoretically required to compress adiabatically from } P_1 \text{ to } P_2}{\text{Work actually done in compressing from } P_1 \text{ to } P_2}$$

is known as the adiabatic efficiency η_a. If the word " isothermally " replaces the word " adiabatically " in the above expression, then the ratio is known as the " isothermal efficiency " η_i.

It should be noted here that the three ratios, T_2/T_1, ρ_2/ρ_1, P_2/P_1,* all increase as the initial temperature decreases for given values of W.†

Losses in the Compressor

The losses normally occurring come under four general headings :—

(*a*) Fluid friction loss.
(*b*) Shock loss.
(*c*) Leakage.
(*d*) Bearing friction.

(*a*) Fluid friction losses occur wherever fluid is moving over a surface, and so the extent of friction loss is governed, among other things, by the area of surface in contact with moving fluid in proportion to the quantity flowing, *i.e.*, it is greater the smaller the " Hydraulic Mean Depth." This particularly refers to portions where the fluid velocity is high. For this reason it is probable that there is a much smaller friction loss in a " vortex " diffuser space than a bladed diffuser system, in spite of the large annular dimension of the former. In the shrouded types the internal friction in the rotor is not great as the relative velocity of the gas over the internal surfaces is low, whereas in the unshrouded types, the fluid engaged by the rotor is travelling at a high velocity over the stationary casing. On the other hand, with the shrouded types there is a disc friction and " pumping " loss between each shroud and the casing.

The energy loss in friction is converted almost directly into heat, and the effect therefore is that of a heat addition during the compression process. It cannot affect the work capacity of the impeller alone, though in the diffuser space the velocity will be reduced, and thus friction may reduce the work capacity of the compressor as a whole.

(*b*) Shock Losses.—These occur mainly at the inlet to the impeller and at the inlet edge of the diffuser blades (where fitted) depending largely on the design.

Since it is impracticable to have a point inlet at the centre, the air entering the inlet eye has to enter the rotor at a radius where the speed of the radial blades is considerable, with the result that there will be a serious shock loss if special measures are not provided to counteract this. Methods of avoiding shock at inlet will be discussed.

The radial velocity at any point in a compressor depends on the capacity, and as the capacity of a compressor (usually expressed in cubic feet of free air per second or per minute) may vary between wide limits, and is only determined by the consumption, the radial component of the velocity may vary, thus the direction of flow varies as the capacity varies.

If fixed diffusers are employed then the condition for no shock is that the direction of flow of the fluid entering the diffusers should be parallel to the leading edge of the diffuser blades. If it is not, then the fluid meets the diffuser with an " angle of attack."

Fixed diffusers can be correct only for one given capacity; at every other capacity there will be a shock loss depending in magnitude on how far from the designed capacity the compressor is working. The result of shock is heat and eddy, the latter ultimately being converted into heat. We see, therefore, that the principal losses result in a heat addition during compression. It is

* It must be remembered here and throughout, that P_1, T_1, etc., are the conditions immediately on entry to the rotor, and owing to the limited dimensions of the impeller inlet, they are often appreciably below those of the atmosphere.

† In other words, for a constant supercharger speed the supercharger compression ratio increases with height, a fact which does not seem to be realised.

clear that the vortex type of diffuser space will be very much better than a bladed diffuser space in so far as freedom from shock is concerned.

(*c*) Leakage losses are most important with multi-stage compressors, though with shrouded types there will be a leakage from the diffuser space to the shaft, through the spaces between the shrouds and casing.

(*d*) Bearing Friction. This is obviously very small.

For multi-stage compressors (shrouded types) Kearton gives the following figures for losses :—

Disc Friction	...	7.23 per cent.
Surface Friction	...	5.2 ,,
Shock Losses	...	5.5 ,,
Leakage ...	...	2.0 ,,
Bearing Friction	...	1.57 ,,

Increase of $\int_{P_1}^{P_2} VdP$ due to heating during compression 1.5 per cent.

He further states that the losses per pound of fluid are proportional to $v_p^{1\cdot6}$ to $v_p^{2\cdot0}$, thus since the work done on the fluid per pound is proportional to $v_p^{2\cdot0}$ the percentage loss should not increase with increase of peripheral speed.

Adiabatic Efficiency.—The effect of the losses is to heat the fluid during compression, and therefore the actual compression curve will be steeper than the adiabatic, and for a given pressure rise the temperature will be higher and the density smaller at the end of compression than if the compression had been adiabatic. This is doubly unfortunate from the supercharging point of view, particularly the reduction of density effect. It should be noted that the increase of $\int_{P_1}^{P_2} VdP$ being very small, the relations between pressure and work capacity given in (5) and (11) above, are substantially unaffected by losses, though final temperature and density are appreciably affected.

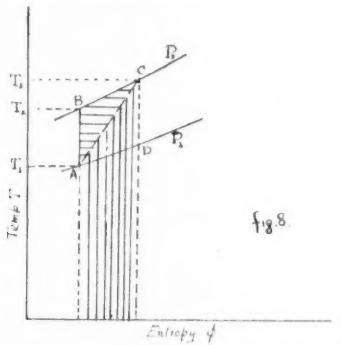


Fig. 8

Referring to the temperature-entropy diagram, Fig. 8, AB represents the adiabatic compression from P_1T_1 to P_2T_2 and AC the actual compression from P_1T_1 to P_2T_3 (shown dotted because only the initial and final conditions may be determined). Then the vertically shaded area represents the work converted into heat during compression, and the horizontally shaded area represents the increase of $\int_{P_1}^{P_2} VdP$ as a result of this, thus the end condition is the same as

though the gas had been compressed adiabatically from P_1 to P_2 and then heated at constant pressure from T_2 to T_3. The total energy required to produce this end condition P_2T_3 is therefore $K_P(T_2-T_1)+K_P(T_3-T_2)$, where K_P is the specific heat at constant pressure in mechanical units. The total energy required to drive the compressor (since there is no heat from any external source) is given by $K_P(T_3-T_1)$ and the work required for ideal adiabatic compression from P_1 to $P_2=K_P(T_2-T_1)$, therefore the adiabatic efficiency is given by

$$\eta_a=(T_2-T_1)/(T_3-T_1) \quad . \quad . \quad . \quad . \quad (12)$$

In words, the actual temperature rise equals the theoretical temperature rise divided by the adiabatic efficiency. Now from (8) we see that the theoretical temperature rise is directly proportional to the work capacity and therefore to the square of the peripheral speed, and therefore that the actual temperature rise is also proportional to the work capacity, *i.e.*,

$$T_3-T_1=0.00298\,(W/\eta_a) \quad . \quad . \quad . \quad . \quad (13)$$

Inlet Types.—The types of inlet for compressors deserve consideration as they affect both the losses and the work capacity of the compressor.

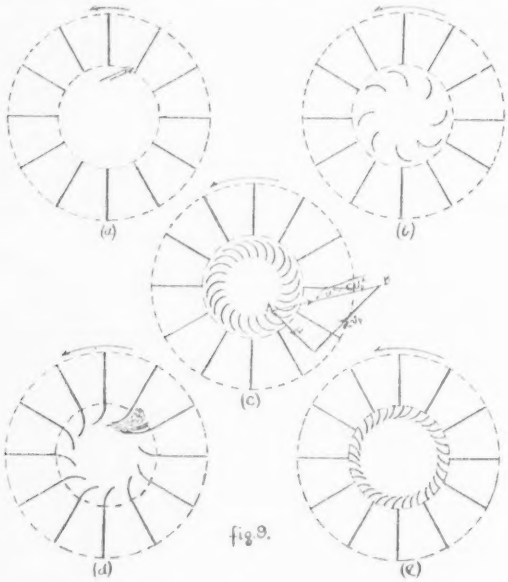


fig 9.

Five types of inlet are shown in Figs. 9*a-e*.

For simplicity, the diagrams are drawn as though the flow at inlet were radial, though in practice the flow through the inlet is usually axial.

9*a* is a type without inlet guide vanes. The relative velocity is shown by arrows. It is clear that there would be a serious shock loss. If the speed of the radial blades at inlet is cv_p where v_p is the peripheral speed and c a constant and equal to the ratio of mean radius of inlet to the maximum radius of impeller, then the work capacity of the impeller $=v_p^2/2g-c^2v_p^2/2g=(v_p^2/2g)(1-c^2)$ and the work capacity of the diffuser system $=\{v_p(1-\pi/n)\}^2/2g=(v_p^2/2g)(1-\pi/n)^2$ from (4); (assuming that the velocity at discharge is very small and roughly equal to the radial velocity at the inlet) the total work capacity per pound

$$=W=(v_p^2/2g)(1-c^2)+(v_p^2/2g)(1-\pi/n)^2=(v_p^2/2g)[1-c^2+(1-\pi/n)^2]$$

Fig. 9*b* shows a type with stationary guide vanes. The function of these vanes is to cause the fluid to acquire the same velocity in a tangential direction, as the portion of the radial blade with which they come in contact. That is, their duty is to increase the velocity of the air from u feet per sec. in a radial direction to a velocity $\sqrt{(u^2+cv_p^2)}$ in a direction AB, this increase of velocity must be accompanied by a drop in pressure. In other words, the stationary guide vanes must act like the nozzle stage of a turbine if they are to perform the required duty. This they are obviously too widely spaced to do, and their actual effect will be to throttle the gas without producing an appreciable change in direction, and therefore without reducing the shock loss. They are therefore worse than no guide vanes at all.

Fig. 9*c* shows a type in which the stationary guide vanes are sufficiently closely pitched to act as nozzles. In this case the pressure energy on entering the impeller $=-c^2v_p^2/2g$, the rise in the impeller $=(v_p^2/2g)(1-c^2)$, therefore the net rise from the atmosphere $=(v_p^2/2g)(1-2c^2)$, this is the work capacity of the impeller. The total work capacity $W=(v_p^2/2g)[1-2c^2+(1+\pi/n)^2]$, *i.e.*, less than in case 9*a*, though the efficiency should be greater owing to the decrease of shock loss.

Fig. 9*d* is a type with rotating curved guide vanes, there being one guide vane per radial blade. Much the same remarks apply to this as to 9*b*, though in this case the function of the vanes is to remove the relative tangential velocity of the entering gas, and since the radial velocity is determined by the capacity, they can only remove this relative velocity without shock and eddy by converting it into pressure; in other words, they must act like rotating diffuser blades. They are clearly too widely pitched to do so, and the entering gas will "stall" over the backs of the guide vanes. 9*e* is a similar type, but in which the guide vanes are pitched closely enough to provide the required effect, *i.e.*, they act like a reversed stage of a Parsons' reaction wheel.

The pressure energy rise in the guide vanes $=c^2v_p^2/2g$, and in the impeller $=(v_p^2/2g)(1-c^2)$, therefore total pressure energy rise in the impeller alone $=v_p^2/2g$ and the total work capacity $W=(v_p^2/2g)[1+(1-\pi/n)^2]$.

This is the type with the largest work capacity for a given peripheral velocity. Probably there is not much difference in the efficiency of the types 9*c* and 9*e*, though since the work capacity of 9*e* is the greater in proportion to the amount of rubbing surface, the efficiency should be slightly higher. In each type the work capacity is seen to be proportional to the square of the peripheral speed, the value of the connecting constant depending on the design. Assuming that the mean radius of inlet is one-third of the maximum radius (it is often more and rarely less), and further assuming that $\pi/n=0.2$ (with 16 blades $\pi/n=0.196$), the value of the constant in $W=kv_p^2/g$* for each type is as follows:—9*a*, 0.765; 9*c*, 0.71; 9*e*, 0.82. Since we have assumed the same diffuser conditions for each type, it is clear that the type of inlet has an important effect on the work capacity of the compressor.

The Characteristic Curves.—We have mentioned above that the capacity of a compressor may vary between wide limits. It has been found that the delivery pressure varies with the capacity, and the curves of delivery pressure plotted against capacity with peripheral speed as parameter are known as the characteristic curves of the compressor. Fig. 10 is an illustration of a family of characteristic curves. The line AB joins the "breakdown" or "surging" point on each curve, to the left of which the characteristic curve is unknown, as the breakdown point is the lowest capacity at which the delivery is steady, if the consumption causes the capacity to drop below this minimum value, then a violent fluctuation in delivery commences, known as "surging." For further information on surging the reader is referred to Kearton's book.

* As v_p^2/g equals the maximum attainable work capacity, the constant k might be termed the "Work Capacity Coefficient."

The reason for the decrease in pressure with increase of capacity is not difficult to see. Increasing capacity means increasing radial velocity, and increasing axial velocity at the inlet, therefore the depression at the inlet increases, thus detracting from the final pressure head; also as the compressor is working further and further from its designed capacity, losses increase rapidly, further reducing the pressure head.

In the case of the supercharger, the delivery volume is governed by the engine, and for constant r.p.m. the delivery volume is approximately constant. For an ideal compressor $V_1/V_2=(1+0.00298\ W/T_1)^{1/(\gamma-1)}$ and therefore if W is constant, the ratio V_1/V_2 varies a little as T varies (*e.g.*, changes in altitude), but the variation of volume of free air taken in will extend only over a small range.

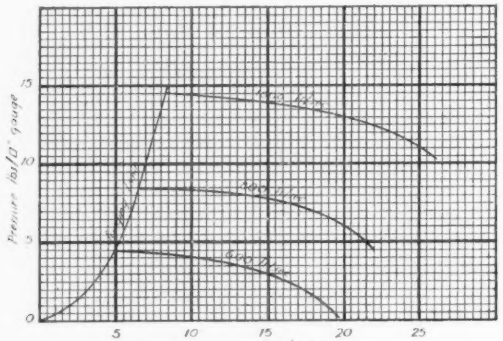


Fig. 10

Supercharging and the Engine

M.E.P. and Inlet Density.—The expression " maintaining sea-level induction pressure " is often used in connection with supercharging, and it is a very misleading phrase, as it is liable to convey the impression that if sea-level pressure is maintained in the induction system, then sea-level power is obtained from the engine. This is only true if the induction temperature is the same as the sea-level induction temperature, that is to say, if the " sea-level induction density " is maintained, since the M.E.P. is directly proportional to the induction density. The object of supercharging is therefore to " maintain sea-level induction density."

Normal sea-level density can vary through a considerable range, the density being a maximum on a very cold day of high barometric pressure and a minimum on a very hot day when the barometer is low; the maximum variation in ground level density of the atmosphere being from 0.0725lbs. per c. ft. to 0.087lbs. per c. ft. Thus an engine developing 100 b.h.p. (mechanical efficiency=88 per cent.) in standard atmosphere (density=0.0782lbs. c. ft. at sea-level) may develop as much as 112.7 b.h.p. on a very cold day, and its b.h.p. may fall to 92 on a very hot day.* Similar variations would occur at heights, and so it becomes necessary to use the standard atmosphere as a basis.

* In actual practice the density will be modified owing to cooling in the carburettor, which will be greater on a hot day than on a cold day—that is to say, the effect of the carburettor will be to reduce this variation in power, but nevertheless it is a well-known fact that a car engine " pulls " better in the cool of the evening than during the day.

Some data on the standard atmosphere are given below:—

Altitude. H	Density. ρ_H	Temperature. T_a	Pressure. P_a	Specific Volume. V_a	ρ_H/ρ_o
0 feet	0.0782lbs./c. ft.	282° Abs.	2,118lbs./sq. ft.	12.8 c. ft./lb.	1.000
10,000 ,,	0.0562 ,,	267° ,,	1,440 ,,	17.79 ,,	0.720
20,000 ,,	0.0407 ,,	247° ,,	965 ,,	24.55 ,,	0.522
30,000 ,,	0.0289 ,,	229° ,,	635 ,,	34.60 ,,	0.370
40,000 ,,	0.0194 ,,	220° ,,	410 ,,	51.50 ,,	0.249

Since M.E.P. is proportional to the induction density $\text{i.h.p.}_H = \rho_H/\rho_o \times \text{i.h.p.}_o$ where i.h.p._H and i.h.p._o are the i.h.p. at height H and sea-level respectively. (In actual fact the ratio i.h.p./ρ should increase slightly with height owing to the improvement of thermal efficiency as a result of the all-round lower temperatures of the thermal cycle.) In practice the b.h.p. falls off more rapidly than density, and the reason for this is simple, though one has never seen the explanation given with the fact in any text-book so far.

I.h.p. − b.h.p. = horse-power absorbed in the engine, and of this horse-power absorbed in overcoming mechanical losses, the greater proportion is constant and independent of the density.

Let $M = M_k + M_v$ where M_k is the constant portion of the mechanical loss and M_v is the remainder which is proportional to the M.E.P., then $\text{b.h.p.}_o = (\text{i.h.p.}_o - M_v) - M_k$ and

$$\text{b.h.p.}_H = (\rho_H/\rho_o)\ (\text{i.h.p.}_o - M_v) - M_k \text{ or } \text{b.h.p.}_H = (\rho_H/\rho_o)\ (\text{b.h.p.}_o + M_k) - M_k \quad (14)$$

The portion of the mechanical loss which remains constant is that part of internal friction due to the inertia of reciprocating parts, valve and other auxiliary drives, etc. For example, piston friction accounts for 70 per cent. of the mechanical loss, and of the total piston friction, 70 per cent. is due to the inertia of the reciprocating parts. In short, the mechanical efficiency decreases with the density until in the limit the density becomes so low that the power developed is just sufficient to turn the engine over against internal losses. It has been assumed in arriving at (14) that the r.p.m. remains constant.

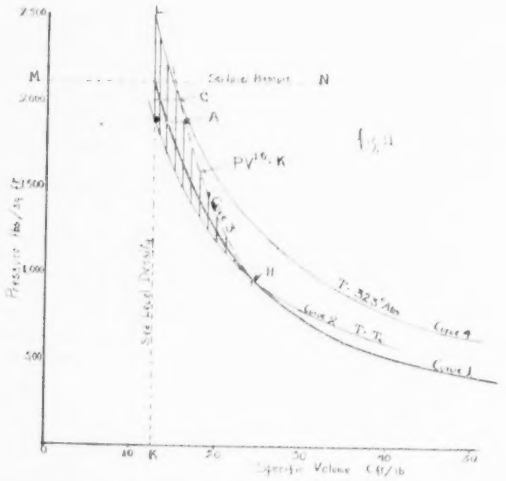
Ricardo makes it quite clear in the section of "The Internal Combustion Engine," Vol. I, dealing with mechanical efficiency, that the mechanical efficiency depends on the M.E.P., but he does not include similar remarks when dealing with the effect of altitude.

The curve from Table 2 in Fig. 13 shows the power altitude curve of an engine of i.h.p. at ground level = 100 and b.h.p. = 88. Of the 12 h.p. absorbed in the engine, 4 h.p. has been assumed to be the variable portion, and the remaining 8 as constant, *i.e.*, substituting in (14) $\text{i.h.p.} = (\rho_H/\rho_o) \times 100$ and $\text{b.h.p.} = (\rho_H/\rho_o)\ (100-4) - 8 = (\rho_H/\rho_o)\ (88+8) - 8$. The curves are based on standard atmosphere. The variation of volumetric efficiency due to the increasing temperature difference between the cylinder walls and the inducted charge with increase in altitude has not been allowed for.

The Object of Supercharging.—The object of supercharging is to increase the induction density up to as near the ground level density as possible within the limits discussed later.

A diagram for making clear this object is shown in Fig. 11; a pressure volume diagram where Curve 1 represents the standard atmosphere, the line KL the line of sea-level density, and Curve 2 the isothermal through a point on Curve 1 representing the height H. Unless refrigerating methods were used Curve 2 represents a limit in that the state of the air at height H cannot be brought to the left of this curve. Thus the ideal method of bringing the state point from H to the line KL would be along the isothermal 2. This is impracticable, and the next best method is to compress to A and cool at constant pressure to the line KL. Since the state point cannot move to the left of Curve 2, the minimum compression pressure would be that where Curve 2 cuts KL. In

practice, cooling to the temperature represented by Curve 2 is impossible, and so the best we can do is to cool at a higher pressure C to the line KL. The higher the pressure, the easier it will be to cool to KL. An inter-cooler is thus seen to be essential to obtain sea-level density in the induction system, without exceeding a pressure represented by the line MN, which represents sea-level pressure (standard atmosphere). If there is no inter-cooling, then the state point representing induction conditions, will be the end point of the compression Curve 3. [Note: These curves are actual plottings; the point H represents 20,000ft. and Curve 2 is the isothermal $T=247^{\circ}$ Abs. The compression Curve 3 is plotted from $PV^{1\cdot6}=K$.]



If an inter-cooler is used, then it becomes necessary to place the supercharger on the air side of the carburettor. This has disadvantages, *e.g.*, the petrol tanks must also be under pressure, and the carburettor must be surrounded by a "box" in such a way that the whole carburettor is placed in an "artificial atmosphere." This will mean leakage where the controls pass into the "box."

Supercharging and Engine Capacity

At ground level and with unsupercharged engines, the swept volume governs the capacity of the engine, because the residual exhaust gas prevents the fresh mixture from occupying the clearance volume (except for the slight effect of the inertia of the outgoing exhaust gas), but with the supercharged engine at heights, the induction pressure is higher than the pressure in the combustion head (virtually that of the outside atmosphere) at the opening of the inlet valve, and a rather complicated process takes place. The incoming gas is throttled as long as the pressure difference exists, the amount of throttling depending on the pressure difference. The pressure difference will be practically equalised after a short interval of time, the incoming gas doing work on the residual gas and also on the portion of the inlet gas which has been throttled. It would be difficult to say what the effect is on the engine capacity as a result of this complicated process, but there would be a definite increase, since some of the inlet gas will occupy space left by the compression of the residual gas. The supercharger therefore compresses the residual gases as well as the aspirated mixture. This increase of capacity will not be taken into consideration when calculating power

curves, as it is an unknown quantity, but it is a point which should be remembered.

Horse-Power Required by the Supercharger

The power required to drive a supercharger is governed by three factors:—

(*a*) The amount of compression.
(*b*) The adiabatic efficiency.
(*c*) The weight of fluid compressed in unit time.

The work per pound per second for the ideal supercharger (*i.e.*, ideal adiabatic compression) is the work capacity W and is obtained from either (5) or (7). The work actually put into the supercharger per lb. per second is the work required in the ideal case divided by the adiabatic efficiency. The weight per second dealt with is governed by the engine capacity, the engine volumetric efficiency and the induction density. The latter coincides with the density at the end of compression when there is no inter-cooling. In general, gross

$$\text{h.p.}_{sc}=(CW\rho_i/550\,\eta_a)\,\eta_v \quad . \quad . \quad . \quad . \quad . \quad (15)$$

where h.p._{sc} = power required to drive the supercharger, C = engine capacity in cubic feet per second, W is the work capacity of the supercharger ($=k\,v_p^2/g$), η_a is the adiabatic efficiency, ρ_i is the induction density in pounds per cubic foot, and η_v = volumetric efficiency.

This is the gross horse-power required, *i.e.*, the power required in the supercharger transmission. However, a large proportion of this power is regained on the induction stroke in the engine, the magnitude of the regain being $(P_i-P_a)\,LAN/550$, where P_i is the induction pressure, P_a is the atmospheric pressure (pounds per sq. ft.), L the stroke in feet, A the piston area in sq. ft., and N the total number of induction strokes per second. The net horse-power required by the supercharger is therefore $(1/550)\,[CW\rho_i\eta_v/\eta_a-(P_i-P_a)\,LAN]$. Now $LAN=C$ therefore net

$$\text{h.p.}_{sc}=(C/550)\,[W\rho_i\eta_v/\eta_a-(P_i-P_a)] \quad . \quad . \quad . \quad (16)$$

Now since $W=\int_{P_1}^{P_a} VdP$ and $W\rho_i=W/V_i$, where V_i is the induction specific volume, then $W\rho_i=(\int_{P_i}^{P_a} VdP)/V_i=P_i-P_a$ very nearly when P_i-P_a is small and there is no cooling, (16) thus becomes

$$(C/550)\,(P_i-P_a)\,(1/\eta_a-1) \quad . \quad . \quad . \quad . \quad (17)$$

Equation (17) would do as a rough approximation for the net horse-power absorbed in an uncooled supercharger for a small degree of supercharge.

The most satisfactory general method of calculating the net b.h.p. of the engine, is to find the gross horse-power required by the supercharger from (15) and subtract it from the gross b.h.p. of the engine when the latter is calculated with the assumption that the M.E.P. has been increased by (P_i-P_a) as a result of supercharging, or in other words, $\text{M.E.P.}=k\rho_i\eta_v+P_i-P_a$, where k is a constant and ρ_i is the induction density.

Taking mechanical losses into consideration

$$\text{b.h.p.}=(1-M_v)\ \text{i.h.p.}-\text{h.p.}_{sc}-M_o$$
$$\text{i.h.p.}=\text{M.E.P.}\times LAN/550=(k\rho_i\eta_v+P_i-P_a)\,(LAN/550)$$
$$\text{b.h.p.}=(1-M_v)\,(k\rho_i\eta_v+P_i-P_a)\,(LAN/550)-CW\rho_i\eta_v/550\,\eta_a-M_o$$
$$=(C/550)\,[(1-M_v)\,(k\rho_i\eta_v+P_i-P_a)-W\rho_i\eta_v/\eta_a]-M_o$$

where M_v is the variable mechanical loss expressed as per cent. of i.h.p., M_o the constant mechanical loss in h.p., h.p._{sc} the gross horse-power required by the

supercharger, P_i and ρ_i the induction pressure and density, P_a the atmospheric pressure, L, A and N the stroke, piston area and total number of working strokes per second.

Limitations to Supercharging

Temperature.—The chief limitation to supercharging is the maximum permissible induction temperature, which Mr. Fedden puts at 50°C., *i.e.*, 323° Abs. If this temperature is exceeded, then the compression temperature will be too high and detonation is liable to occur. One feels that the expression " a large degree of detonation " should be used where the word " detonation " is normally used, because there cannot be a sudden change from no detonation at all to complete detonation. Detonation is ignition produced by the heat of compression, and therefore spreads with the speed of a compression wave. It is probable that some degree of detonation is always occurring in a normal engine at full throttle, according to the following reasoning:—Any portion of an inflammable mixture will ignite when that portion is raised to its ignition temperature, and in normal flame propagation each portion of the mixture is raised to its ignition temperature by the heat of the adjacent burning portions.

In an unconfined mixture, the sole temperature raising factor present is the heat of the burning portion, and the rate of flame travel will depend on the turbulence, strength, and initial temperature of the mixture. If each portion has to be raised through a large temperature, then the time necessary to do it will be relatively long, and the rate of flame propagation correspondingly slow. If, however, the mixture is confined to a given volume, then there are two temperature raising factors present, namely, the heat from the burning portion and the rise in pressure, which will produce a rise in temperature of the unburnt portion, and if the initial temperature was not far from the ignition temperature in the first place, the unburnt portion may reach its ignition temperature as a result of the pressure rise alone, and partial detonation will take place.

The detonation may not appear to an observer as such, due to the shock-absorbing effect of the portion which has already burned before detonation of the remainder occurs. The fact that there is some degree of detonation in the combustion process in normal engines may prove to be part of the explanation of the rapidity with which combustion is known to take place. Any car owner knows that " knock " can be varied in violence by varying the ignition or throttle.*

Taking 323° Abs. as the upper limit of induction temperature then we may add Curve 4 in Fig. 11 as another limit. Thus, the supercharging process commencing at H must end within the vertically shaded area. If there is no inter-cooling, the limit of supercharging is marked by the intersection of the compression curve through H and the isothermal $T=323$. The maximum temperature rise which may take place is given by $323-T_a$ where T_a is the absolute temperature at the supercharger inlet at the height H.

With inter-cooling, the temperature at the end of the whole supercharging process must equal or be less than 323. It is clear from Fig. 11 that the amount of supercharging and the amount of inter-cooling are interdependent, the greater the inter-cooling, the smaller the amount of supercharging to obtain a given density, always provided that the final state point is to the right of isothermal T_a.

Density.—Another limit to supercharging is the maximum permissible pressure in the cylinder, and this can be shown to be very nearly proportional to the induction density. For combustion at constant volume

$$P_m/P_e=P_m/P_i r^n=T_m/T_e=T_m/T_i r^{n-1} \quad (n=\text{compression exponent})$$

* Throttling does not reduce the compression temperature, but it reduces the compression density and also results in an increased proportion of residual gases in the burning mixture, both factors reducing the rate of flame travel.

therefore

$$P_m = (P_i/T_i)\, rT_m = R\rho_i rT_m \quad (R \text{ is the gas constant})$$

where P_m and T_m are the maximum pressure and temperature, P_e and T_e are the compression pressure and temperature, P_i, T_i and ρ_i are the induction pressure, temperature and density, and r is the compression ratio. Thus the maximum pressure is proportional to the induction density and the maximum temperature, and is independent of the induction pressure alone. Since the maximum temperature is something of the order of $T_e + 1{,}600$, the percentage variation of T_m for variations in T_i will be very small, so that it is sufficiently true to say that the maximum pressure is proportional to the induction density, and therefore to ensure that the maximum pressure is not exceeded, a limit should be placed on the induction density. It may be seen that the greatest loads occur in a normal engine on very cold dry days, and that on hot days the induction pressure could well exceed the normal atmospheric pressure without overloading the engine. It is rather curious, therefore, that a boost gauge should be used as a guide to the supercharging limit.

From (7) it is clear that the temperature rise in a supercharger is governed only by its peripheral speed and efficiency, and is independent of the pressure either at inlet or outlet, and therefore a boost gauge in no way indicates whether the temperature of the induction system is exceeding the limit. Experience bears out the fact that the limit prescribed by the boost gauge does not preserve the engine from detonation. The writer confesses to having " gone through the gate " on the climb (1,500 r.p.m.) in a Siskin IIIA (supercharged Jaguar) without noticing any detonation, whereas when diving (r.p.m. 1,800) with the engine throttled to below " Maximum Permissible Boost," a certain degree of detonation has been perceived. It is evident that the limit should be indicated on the tachometer and not on a boost gauge; again, the limit could vary with the temperature of the atmosphere, higher r.p.m. being permissible on a cold day than on a hot day, without risk to the engine. The " Wow " of a diving Siskin is well known, and observers may have noted that though both the unsupercharged and supercharged Siskins make a colossal noise (largely due to the airscrew, no doubt) the supercharged Siskin seems to have a " harder " note; in fact, it seems to vibrate the ground. This " hardness " is due to detonation, in the opinion of the writer.

Methods of Drive

Possible methods of driving superchargers may be divided into two general headings: Driven by the engine, and Independently driven. These may be sub-classified as follows:—

A. Engine Driven.
  (*a*) Exhaust driven.
    (1) Exhaust Turbine.
    (2) Exhaust-heated vapour engine.
  (*b*) Gear Transmission.
    (1) Single gear ratio with permanent coupling—throttle control.
    (2) Single gear ratio with clutch.
    (3) Single gear ratio and by-pass.
    (4) Single gear ratio and variable inlet valve timing.
    (5) Multi geared and clutch.
    (6) Infinitely variable friction gear.
  (*c*) Indirect transmission.
    (1) Electric.
    (2) Fluid transmission.

B. Independently Driven.
  (*a*) Auxiliary engine.
  (*b*) Wind-driven turbine or fan.

Of these, the following are impracticable on the score of weight :—(*a*) (2), (*c*) (1) and (2). Practical difficulties will not permit the use of methods (*b*) (3), (4) and (6); others present difficulties but cannot be called impracticable.

(*a*) (1). Exhaust-driven turbine—a method which has been tried in several countries, and found very satisfactory from the supercharging point of view, but which has been abandoned at present in this country owing to the difficulties which have arisen from the very high temperatures of the gases in the turbine nozzle box. It is said that several fires in the air have occurred in France through the use of exhaust-driven turbo-superchargers.

(*b*) (1). Single gear ratio and permanent coupling with throttle control—a method which has been in general use for some time. There are many evils in the system. Below a certain height the engine has to be throttled, *i.e.*, the pressure of the induction system is kept down to atmospheric while the temperature is well above normal, *i.e.*, the density is low, and also the r.p.m. are well above the normal for the engine, therefore the ground-level power is well below the figure it would be if the engine were normally aspirated. Above that height the full degree of supercharging cannot be employed as the maximum speed of the engine governs the maximum speed of the supercharger.

(*b*) (2). A similar method, but a clutch is used to throw out the supercharger and thus convert the engine to normal aspiration. This has a disadvantage in that the supercharger usually also performs the duty of an induction fan in producing correct distribution, so that by uncoupling the supercharger, the induction fan effect is also lost.

(*b*) (5). A type in which the transmission is through a two or three speed gear and clutch. It would mean weight, but should prove a fairly satisfactory method of control.

Of the " independently driven " methods, method (*b*) involves placing gadgets in the air stream. Strictly speaking, this cannot be called an independent drive, since the drive is paid for in drag, which in turn is overcome by the engine and airscrew, and therefore if the main airscrew were 75 per cent. efficient and the impeller and gearing 60 per cent. efficient (probably a very high figure for the latter), the overall efficiency of the " transmission "=45 per cent. For this reason alone this method must be regarded as a " wash-out." This applies to any wind-driven auxiliary.

An independent engine relieves the main engine of the task of driving the supercharger altogether, but when not in use, *e.g.*, near the ground, it is the addition of so much dead weight without any increase in power. However, as height is gained and the auxiliary engine begins to supercharge, a large proportion of the horse-power put into supercharging is regained in the main engine as we have shown before, until eventually a height is reached when the auxiliary engine is working at full power, and the whole system is equivalent to a single engine of greater rating than the main engine, with a gear-driven supercharger.

Therefore, for the purpose of comparison, a 90 h.p. main engine with 10 h.p. auxiliary should be compared with a 100 h.p. engine having a supercharger driven by the main engine. The weight per horse-power of the two power plants should not differ much at the supercharged height, though the 90 + 10 system would probably be a little heavier as small engines appear to have a larger weight per horse-power than large ones, though one does not regard this as an immutable rule.

This system has many advantages, though it starts off with the disadvantage that the supercharger must be placed on the air side of the carburettor of the main engine :—

(*a*) It may be applied to any standard unsupercharged engine with very little modification.

(*b*) The degree of supercharging is easily controllable.

(*c*) May be placed to suit the location of an inter-cooler in the leading edges of the planes.
(*d*) May drive other auxiliaries, such as generators, etc.
(*e*) May be used to "supercharge" a hermetically sealed cabin (in which case the supercharged air might pass through the cabin on its way to the carburettor of the main engine).
(*f*) One supercharger may feed several engines.
(*g*) Makes possible the supercharging of engines in tropical countries.

In addition to the slight disadvantage mentioned above, another disadvantage is that air-cooling would mean extra drag, though how much greater the drag of an air-cooled 90–10 h.p. system would be than an air-cooled 100 h.p. engine it is difficult to say. (If an efficient inter-cooler is used, it might pay to place the auxiliary engine out of the airstream and compel the air inspired by the supercharger to pass round and cool the cylinders of the auxiliary engine. However, this is obviously a bad principle, since it further reduces the density of the inspired air.)

Comparison of Maximum Power with Height for Various Types of Drive

Type (*a*) A supercharger driven by the engine, but the speed of which is variable independently of the engine, no inter-cooling. This corresponds to an infinitely variable gear transmission.

The limits are (*a*) The induction temperature must not exceed 323° Abs.
(*b*) The induction density must not exceed 0.0782lbs./c. ft. (Though we have seen that in practice the induction density may rise to 0.087lbs./c. ft. in normal engines.)

The maximum temperature rise which may take place is thus fixed at $323-T_a$, where T_a is the atmospheric temperature. If T_t is the corresponding temperature of compression for the same pressure rise with adiabatic compression then $(T_t-T_a)/(323-T_a)=\eta_a$ and from this we can obtain T_t. Now with adiabatic compression the density of the compressed gas ρ_t may be obtained from $\rho_t/\rho_a=(T_t/T_a)^{1/(\gamma-1)}$ and the induction density $\rho_i=(T_i/323)\,\rho_t$, therefore $\rho_i=(T_i/323)\,(T_t/T_a)^{1/(\gamma-1)}\,\rho_a$. From this we may obtain the weight of mixture aspirated per second. From $P_i=\rho_i R T_i$ we obtain the induction pressure and this enables us to calculate the regain in the engine of the h.p. expended on the supercharger.

Fig. 12 shows induction densities plotted against altitude for various adiabatic efficiencies, the compression temperature being 323° Abs. The heights where these curves cross the line $\rho=0.0782$ are the heights below which the degree of supercharging is limited by the induction density, and above which the sole limit is the induction temperature.

The general formula for the b.h.p. of an engine fitted with an uncooled supercharger is:—

$$\text{b.h.p.}=(C/550)\,[\,1-M_v)\,(778\,f\,.\,h\,.\,\eta_t\eta_v)\,\rho_i+(P_i-P_a) - \eta_v\rho_i R\,\{\,\gamma/(\gamma-1)\,\}\,(T_i-T_a)\,]-M_k \quad . \quad . \quad . \quad . \quad (18)$$

Note.—The above formula (18) assumes constant r.p.m., otherwise M_k varies with the r.p.m.

C = capacity cubic ft. per sec. = LAN.
η_t = indicated thermal efficiency.
η_v = volumetric efficiency.
f = fuel/air ratio by weight.
h = calorific value of fuel B.T.U./lb.

Taking $f=1/15$, $h=10{,}000$ B.T.U./lb., $\gamma=1.4$, $R=96$, then (18) becomes:—

$$\text{b.h.p.}=(C/550)\,[\,(1-M_v)\,(986{,}000\,\eta_t\eta_v)\,\rho_i+(P_i-P_a)-336\,\eta_v\rho_i\,(T_i-T_a)\,]-M_k$$

ρ_i may be obtained from:—

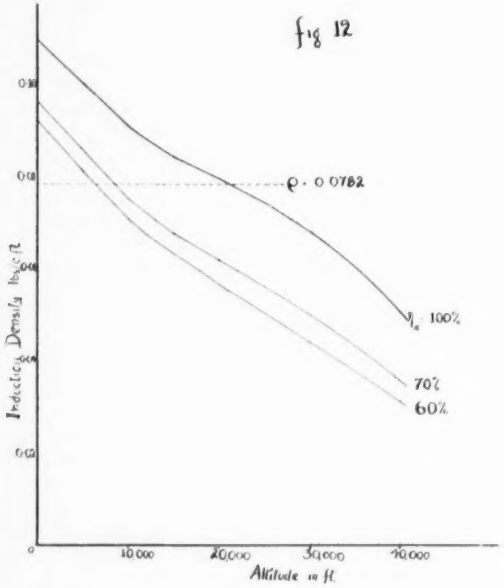
$$\rho_i=(\rho_a/T_a^{2\cdot5}T_i)\,[\,\eta_a\,(T_i-T_a)+T_a\,]^{2\cdot5} \quad . \quad . \quad . \quad (19)$$

Formula (18) will do for a case where inter-cooling is used if the compression temperature (*i.e.*, before inter-cooling) is substituted for T_1. The value of ρ_1 may now be found by multiplying the compression density found from (19) by the ratio T_c/T_1 where T_c is the compression temperature.

Since a formula alone cannot always clearly indicate important facts the tables below may prove useful coupled with the curves in Fig. 13. The following assumptions were made. Ground level b.h.p. = 88; ground level i.h.p. = 100. Of the 12 h.p. mechanical loss at ground level 8 h.p. has been assumed constant and independent of the M.E.P.

$\eta_i = 29.5$ per cent.; $\eta_v = 100$ per cent., limiting induction conditions:— ρ_1 max. = 0.0782; T_1 max. = 323° Abs. Capacity $C = 2.42$ cubic feet per second. Formula (18) is therefore used in the form

$$\text{b.h.p.} = 0.0044\ [278{,}000\rho + P_1 - P_a - 336\ \rho_1\ (T_1 - T_a)] - 8.$$



In the curves the h.p. is expressed as a percentage of the ground level b.h.p.

Case 1. Independently variable supercharger used to give the maximum degree of supercharging at all altitudes. No inter-cooling. Adiabatic efficiency of compressor = 60 per cent.

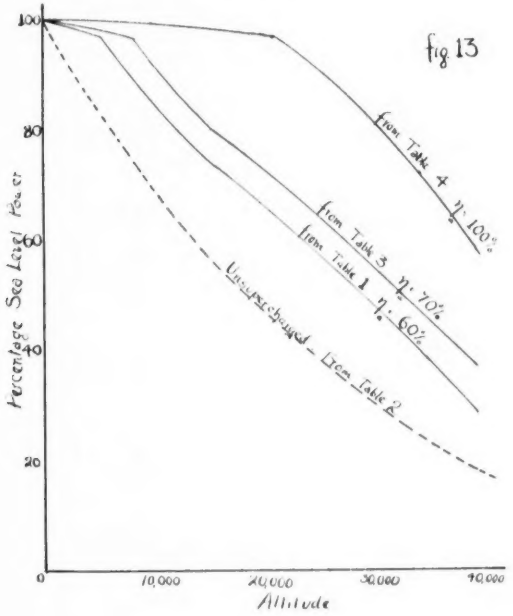
TABLE 1.

H.	Induction Density ρ_1 lbs./c. ft.	Induction Temp. T_1 °C. Abs.	Induction Pressure P_1 lbs./sq. ft.	Gross b.h.p.	Net b.h.p.	Net b.h.p./ 88 %.	Overall Efficiency %.
0	0.0782	282	2118	88.00	80.0	100.00	26.0
5,000	0.0782	315	2360	90.68	86.0	97.75	25.4
10,000	0.0706	323	2190	81.96	75.5	85.75	24.7
20,000	0.0565	323	1750	64.84	58.5	66.50	23.9
30,000	0.0443	323	1372	49.56	43.4	49.40	22.6
40,000	0.0314	323	973	32.98	28.2	28.20	20.7

The figures for the supercharger, on the assumption that the work capacity coefficient is 0.7, are given below:—

TABLE 1a.

H.	Gross $h.p._{sc}$	Gross $h.p._{sc}$/ Gross b.h.p. %	Net $h.p._{sc}$	Peripheral Speed. V_p
0	0.00	0.00	0.00	0
5,000	4.68	5.17	2.04	610
10,000	6.46	7.89	3.16	720
20,000	6.34	9.78	2.89	840
30,000	6.16	12.42	2.92	933
40,000	4.78	14.5	2.30	9.54



For comparison the figures of the equivalent unsupercharged engine are given below:—

TABLE 2.

H.	b.h.p.	Overall Efficiency.	b.h.p. %.
0	88.0	26.00	100
5,000	74.2	25.60	84.3
10,000	61.2	25.10	69.5
20,000	42.1	23.82	47.9
30,000	27.5	21.90	31.25
40,000	15.9	18.86	18.1

Case 2. Independently variable supercharger used to give the maximum degree of supercharging at all altitudes. No inter-cooling. Adiabatic efficiency of compressor —70 per cent.

TABLE 3.

H.	ρ_1	T_1	P_1	Gross b.h.p.	Net b.h.p.	Net b.h.p. %	Overall Efficiency %.
0	0.0782	282	2118	88.00	88.0	100	26.00
5,000	0.0782	305.2	2288	90.15	86.6	98.5	25.55
10,000	0.0750	323	2322	87.60	81.4	92.5	25.04
20,000	0.0618	323	1915	71.85	64.9	73.7	24.25
30,000	0.0496	323	1537	56.68	49.8	56.6	23.18
40,000	0.0356	323	1103	38.62	33.2	37.75	21.50

The corresponding figures for the compressor are given below :—

TABLE 3a.

H.	Gross h.p.$_{sc}$	Gross h.p.$_{sc}$/ Gross b.h.p.%	Net h.p.$_{sc}$	Peripheral Speed.
0	0.00	0.00	0.00	0
5,000	3.55	3.94	1.23	576
10,000	6.2	7.08	2.32	780
20,000	6.95	9.68	2.77	910
30,000	6.88	12.14	2.91	1,010
40,000	5.42	14.02	2.37	1,060

Case 3. As for Cases 1 and 2, but adiabatic efficiency = 100 per cent.

TABLE 4.

H.	ρ_1	T_1	P_1	Gross b.h.p.	Net b.h.p.	Net b.h.p./ 88 %	Overall Efficiency
0	.0782	282	2118	88.00	88.00	100.0	26.00
5,000	.0782	292	2192	89.62	87.60	99.5	25.86
10,000	.0782	304.6	2285	91.74	87.40	99.3	25.71
20,000	.0782	322	2415	94.27	85.60	97.3	25.25
30,000	.0682	323	2113	82.08	72.60	82.5	24.53
40,000	.0508	323	1574	59.34	51.6	58.6	23.47

The corresponding compressor figures are given in Table 4a below.

TABLE 4a.

H.	Gross h.p.$_{sc}$	Gross h.p.$_{sc}$/ Gross b.h.p. %	Net h.p.$_{sc}$	Peripheral Speed.
0	0.00	0.00	0.00	0
5,000	2.02	2.25	0.12	522
10,000	4.34	4.73	0.62	762
20,000	8.67	9.20	2.29	1,078
30,000	9.48	11.53	2.98	1,210
40,000	7.74	13.05	2.62	1,265

A large amount of information may be derived from the figures in the above tables. The most striking feature is the enormous importance of the adiabatic efficiency of the compressor when there is no inter-cooling.

Another fact of importance is that to achieve the limit of supercharging at any height, the peripheral speed of the compressor must be greater the greater the adiabatic efficiency, in fact, if the adiabatic efficiency is raised from η_1 to η_2 then the peripheral speed may be raised from v_1 to $v_1\sqrt{(\eta_2/\eta_1)}$.

It will be noticed that in the above tables the induction pressure is considerably above the normal sea-level pressure (2,118lbs./sq. ft.) for the lower altitudes.

As we have previously argued, as long as the induction density is normal, the maximum combustion pressure will be practically normal, and so it is not necessary to restrict the induction pressure more than to prevent the density from exceeding the normal maximum. If the induction density remains constant, then the induction pressure must be proportional to the absolute induction temperature, *i.e.*, the greater the induction temperature, the greater the permissible induction pressure. It is not possible at present to say how a pilot is going to know what the induction density is at any time, but when the necessity is realised,

no doubt an instrument will be evolved to fill the need. Such an instrument will probably be based on the fact that temperature rise in the supercharging process is proportional to the square of the supercharger r.p.m., and therefore induction temperature = atmospheric temperature + k (r.p.m.)^2 and therefore for constant density, induction pressure, supercharger r.p.m. and external atmospheric temperature are connected by the relation $P_1 = R(T_a + k\,(\text{r.p.m.})^2)$.

Another interesting fact which emerges from the tables is that a supercharged engine has a greater brake thermal efficiency than the unsupercharged engine, above quite moderate heights, *e.g.*, at 10,000 and above when the compressor efficiency = 70 per cent. This is due to the beneficial effect of the higher M.E.P. on the mechanical efficiency. It should be remembered that no allowance has been made for the increase of volumetric efficiency with height in the supercharged cases, and for the improvement of indicated thermal efficiency with height in the unsupercharged case.

The sharp bend in each supercharging curve occurs at the height below which the density limits the supercharging, and above which the temperature is the limit.

It will be appreciated that if an inter-cooler were installed in the above cases, the induction density would be increased, and therefore both the b.h.p. of the engine and the h.p. required to drive the supercharger would be increased, but the latter would increase more in proportion than the former, because the percentage regained in the engine is less, *i.e.*, the percentage of the gross b.h.p. required to drive the supercharger increases, resulting in a decrease of brake thermal efficiency.

Type (*b*). (1) Engine having a supercharger with a single gear ratio such that the supercharged height is 10,000ft. No inter-cooling. Adiabatic efficiency of supercharger = 70 per cent.

The power curve in this case will touch the power curve obtained in Case 2 at 10,000ft., and will lie below it at all other heights. The assumptions are the same as before, with the additional assumptions that above 10,000 the engine speed, and therefore the supercharger speed, remains constant, and below this height the speed is reduced in order that the induction temperature $T_1 = 323°$ Abs. is not exceeded, and the engine is throttled in order that the induction density shall not exceed 0.0782. The figures are given below.

TABLE 5.

H	b.h.p.	b.h.p./88 %	ρ_1	T_1	P_1	r.p.m./Max. r.p.m. %	Brake Thermal Efficiency %
0	72	81.8	0.0782	323	2420	85.4	24.90
5,000	79	89.8	0.0782	323	2420	92.9	25.13
10,000	81.5	92.5	0.0750	323	2323	100	25.02
20,000	58.8	66.8	0.0559	303	1623	100	24.25
30,000	40.5	46.1	0.0406	285	1110	100	23.00
40,000	24.8	28.2	0.0275	276	728	100	20.83

The power required by supercharger, and the peripheral speeds are as follows :—

TABLE 5a.

H	Peripheral Speed.	Gross Power Required.
0	665 ft./sec.	4.04
5,000	724 ,,	5.20
10,000	780 ,,	6.20
20,000	780 ,,	4.63
30,000	780 ,,	3.36
40,000	780 ,,	2.28

Case b(2). As for Case b(1), but supercharged height = 20,000ft.

TABLE 6.

H	b.h.p.	b.h.p./88 %	ρ_i	T_i	P_i	r.p.m/Max. r.p.m. %	Brake Thermal Efficiency %
0	61.75	70.25	.0782	323	2420	73.1	24.92
5,000	67.70	77.00	.0782	323	2420	79.6	25.10
10,000	69.90	79.50	.0750	323	2323	85.7	25.10
20,000	64.90	73.80	.0618	323	1915	100	24.25
30,000	45.40	51.60	.0453	305	1326	100	23.14
40,000	28.50	32.40	.0309	296	878	100	21.30

TABLE 6a (Supercharger).

H	Peripheral Speed.	Gross Power Required.
0	665	3.46
5,000	724	4.46
10,000	780	5.32
20,000	910	6.94
30,000	910	5.09
40,000	910	3.47

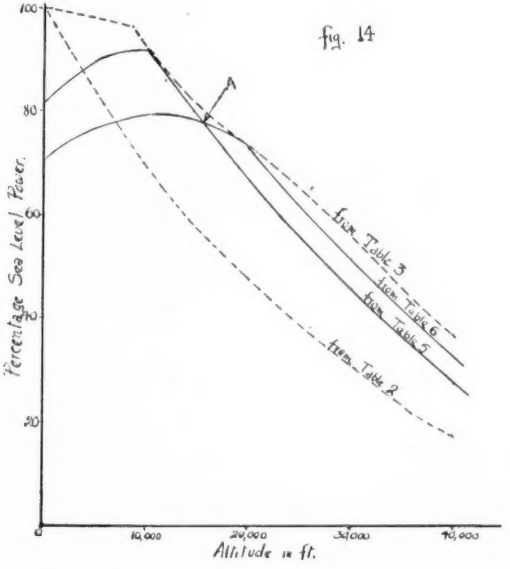
Curves from Tables 5 and 6 are produced in Fig. 14, which shows also the curves from Table 2 and Table 3 reproduced for comparison. The disadvantages of the supercharger with a single gear ratio are obvious, and the loss in ground-level horse-power is seen to be more and more serious the greater the supercharged height. If the engine is throttled to normal induction pressure then the ground-level power will be even less than that shown. For the purpose of this paper, the supercharged height is defined as: "The lowest height at which the engine may run at normal r.p.m. without the induction temperature exceeding 232°Abs." This definition differs from one which is often quoted, namely: "The maximum height at which normal sea-level induction pressure may be maintained." However, the subject of supercharging is comparatively new, and for this reason lacks standardisation in definitions. The "pressure" definition will disappear when it is more realised that sea-level induction density is the aim of supercharging.

Multi-Gear Supercharger.—If two or more gear ratios are provided for the supercharger, then there is a "supercharged height" for each gear ratio, and the height at which the gear change should take place is the height at which the power curves for each gear ratio alone intersect. For example, Fig. 14 could represent the power curves for each gear ratio of a two-gear supercharger, in which case the best height for the gear change to take place would be that at the point of intersection A.

Supercharger Driven by an Auxiliary Engine

Tables 1a, 2a, etc., have columns showing the horse-power required to drive the supercharger expressed as a percentage of the gross b.h.p. From these columns some idea may be obtained of the power required for an auxiliary engine, assuming that the auxiliary engine is itself supercharged. For simplicity it would be desirable to have a constant gear ratio between the auxiliary engine and the supercharger, and therefore it will be necessary to fix a height at which the auxiliary will be developing its maximum r.p.m.; its power will have to be such that it can maintain this speed up to a given height. For example, if 20,000ft. is the altitude at which the auxiliary will be developing maximum r.p.m., then the gear ratio will be such as to give a peripheral speed of supercharger of 910ft. per second. At 20,000ft. the power required to drive the supercharger is 9.68 per cent. of gross b.h.p., and at 40,000ft. with the same compressor speed, the power required is 10.85 per cent. of the gross b.h.p., this therefore represents

the maximum condition. Thus the equivalent power plant with auxiliary engine for comparison with the normal supercharged engine of 100 b.h.p. would be, main engine: 89.15 b.h.p.; auxiliary: 10.85 b.h.p. The ground-level b.h.p. of the system would then be 89.15 per cent. of the equivalent 100 b.h.p. unsupercharged engine. This is a very much better figure for ground-level power than those given in either Table 5 or 6. The power curve will approach the power curve from Table 3, touching it at about 20,000ft., above which it will follow the power curve from Table 6 very closely. An auxiliary engine would therefore seem to be much more satisfactory from the supercharging point of view than gearing. It should be noted that with a multiple-engined plant, one auxiliary engine and one supercharger could feed all the main engines, with a probable reduction in weight.



Future Development of the Turbo-Compressor

We may expect the turbo-compressor to show a steady improvement as time goes on, in three ways:—

(*a*) Increase in maximum permissible speed due to improvements in construction.

(*b*) Improved adiabatic efficiencies.

(*c*) Improved work capacity coefficient.

(*a*) Peripheral speed:—The higher the peripheral speeds that may be used in practice, the greater the work capacity of a single stage compressor. The writer believes that if suitable rotor construction is used, a single stage compressor should suffice for all supercharging purposes. The strongest type in rotation is the unshrouded type, but on some engines it has been found that fracture of the tips has occurred through tip vibration, but one believes there is a simple explanation for the failure of the unshrouded rotor which is placed on the engine side of the carburettor. During bench tests of engines, lumps of ice have been known to fly out of the exhaust, the probable reason being that a coat of ice forms in the induction system, and eventually becoming detached, is sucked into

the engine. It is clear that if this happened with a supercharged engine, the ice must pass through the supercharger, in which case it will " bounce " along the rotor blades, setting up vibration which may result in failure. It seems probable that there would be less risk of this if the supercharger were placed on the intake side of the carburettor. The enormous speeds of which the unshrouded type is capable is shown by the fact that a Ratean unshrouded rotor has been tested up to speeds of 2,100 to 2,200 ft. per sec. One imagines that the unshrouded type may be stiffened as any beam may be stiffened, *i.e.*, by flanging the blades.

(*b*) Improved adiabatic efficiencies :—Improvements in adiabatic efficiency should take place when the need for the improvement is fully realised. At present adiabatic efficiencies of superchargers are below those of commercial compressors, probably due to scale effect to some extent, but also, no doubt, due to lack of experience in compressor design.

The greatest scope for improvement lies in the inlet design and diffuser design, where one believes the greatest losses occur in present types. The vortex type diffuser is likely to supersede all other types in spite of the larger overall dimensions necessary.

(*c*) Improvement of work capacity coefficient:—We have shown that improvement in the work capacity coefficient will mean a greater work capacity for a given peripheral speed, and that this coefficient depends mainly on the inlet design and the number of radial blades. It is unlikely that the latter will exceed 18 to 20 in number, as increase in the number of blades increases the friction loss, but there is still room for considerable improvement in the inlet design.

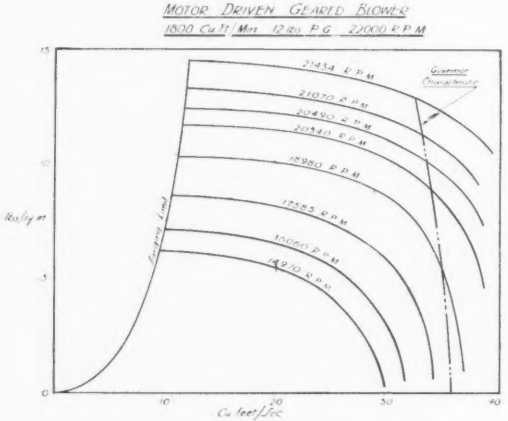


FIG. 15.

Summary and Conclusion

The lines of development which compressor design should follow have been indicated above, and also the possible development of supercharger control.

Next to the exhaust turbine-driven type it appears that an auxiliary engine for the supercharger would be the most satisfactory method of drive and control, provided that practical difficulties are not too great.

Another method, better than the single gear ratio, is a two or three-speed gear and gear changing mechanism.

The adiabatic efficiency of the compressor has a very pronounced effect on the permissible degree of supercharging with uncooled types, and on the degree of inter-cooling necessary with types where inter-coolers are fitted.

The brake thermal efficiency of the unsupercharged engine at heights is increased by supercharging without intercooling, and may either be increased or decreased by inter-cooling, according to the degree of supercharging and intercooling employed. This improvement in efficiency is mainly due to the beneficial effect of the higher m.e.p. on the mechanical efficiency, and partly due to the regain during the induction process of power expended in supercharging.

The restriction to a maximum boost pressure used on modern types of supercharged engines is unsatisfactory and should be replaced by restrictions on the r.p.m. and induction density; the external atmospheric temperature should also be taken into consideration.

Finally, the writer believes it well within the bounds of possibility to fit commercial aircraft with single stage superchargers to enable them to fly economically over long distances at altitudes of the order of 38,000ft., provided that some serious effort is made to improve the efficiency of the supercharger and the method of control.

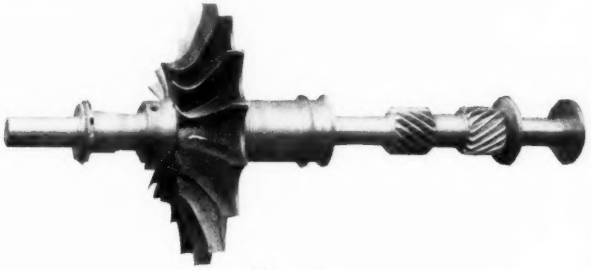


FIG. 16.

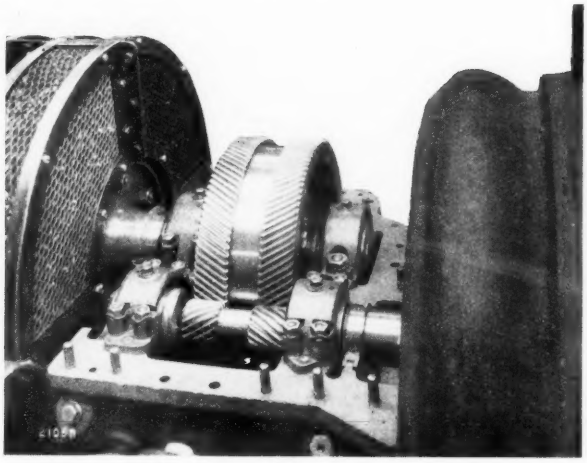


FIG. 17.

APPENDIX

The following is based on information kindly supplied by the British Thompson Houston Co., Ltd., who include the manufacture of turbo-compressors among their many activities.

Fig. 15 shows the characteristic curves obtained from a single stage compressor, the impeller of which is illustrated in Fig. 16.

The compressor was designed for 1,800 cubic feet of free air per minute (14.7lbs./sq. ins. 60°F.) with a delivery pressure of 12lbs. per sq. in. gauge. It was geared to an electric motor, the gearing used being a B.T.H. double helical gear capable of increasing the speed from the 3,000 r.p.m. of the motor to 22,000 r.p.m. The gearing is shown in Fig. 17.

The diffuser space was fitted with discharge vanes.

The diameter of the impeller was 11 inches, and, as may be seen, the radial blades have curved extensions at the inlet portion to act as rotating curved guide blades. It is an example of the type of inlet shown in Fig. 9(*d*).

This compressor was designed for a purpose rather remote from supercharging, namely, the conveying of clay along compressed air pipe lines, and thus the construction was not limited by weight, but, as may be seen from the above figures, it is of a size suitable for the supercharging of large aero engines.

ABSTRACTS AND NOTICES FROM THE SCIENTIFIC AND TECHNICAL PRESS

PHYSICS AND ENGINEERING SCIENCE

Issued by the

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(Prepared by R.T.P.)

No. 20. SEPTEMBER, 1931

Aircraft Design

Rolling Moments due to Rolling and Yaw. (M. Knight and C. J. Wenzinger, N.A.C.A. Rept. No. 379, May, 1931.) (5.1/22001 U.S.A.)

Rotation and torque tests were made of four wing systems rotated at various angles of yaw with different angular velocities. An important parameter is:

Angular velocity × span/flying speed.

The torque is measured by a dynamometer enclosed in a torpedo-shaped casing and is shown with other details of the installation in photographs. Rolling moments are plotted against the parameter for different angles of yaw and are tabulated numerically. Twelve references are given.

Moments of Inertia of Aeroplanes. (M. P. Miller and H. A. Soulé, N.A.C.A. Tech. Note No. 375, May, 1931.) (5.10/22002 U.S.A.)

The moments of inertia of all flight research aeroplanes are measured as a matter of policy. The principal axes of the ellipsoid of inertia are given for ten aeroplanes.

In the plane of symmetry the principal axes coincide with the trim-axis in two cases and are inclined at angles varying from $+2°30'$ to $-3°40'$ in extreme cases. Non-dimensional inertia coefficients are given for comparison of aeroplanes differing widely in dimensions. The use of units is inconsistent.

Pressure Distribution over Fuselage of Pursuit Aeroplane in Flight. (R. V. Rhode and E. E. Lundquist, N.A.C.A. Report No. 380, May, 1931.) (5.10/22003 U.S.A.)

Photographs exhibit the machine with the installation of manometer leads and distribution of orifices over the body. The results are plotted graphically in twenty-three sets of curves and numerically in tables.

Lift Distribution on a Wing of Arbitrary Plan. (I. Lotz, Z.F.M., Vol. 22, No. 7, April 14th, 1931, pp. 189-195.) (5.20/22004 Germany.)

The problem reduces to the solution of an integral equation. Incomplete solutions by Betz, Fuchs, Trefftz, Munk, Glauert and Gates are briefly discussed. The outstanding difficulty lies in the indeterminate value of the integral at the wing tips. The author investigates the possibility of completing the solution, including the determination of rolling and yawing moments, without prohibitive computation.

A result is obtained for a wing with an arbitrary plan, and variable incidence, which gives sufficient numerical convergence.

Expressions are found for rolling and yawing moments and are compared with experimental results.

Fuselage Interference Effect. (A. S. Hartshorn, Airc. Eng., Vol. 3, No. 30, August, 1931, pp. 201-203.) (5.11/22005 Great Britain.)

A number of experimental results are taken from international laboratory tests and are discussed in reference to the aerodynamical qualities of complete aeroplanes. The results indicate some advantage for a high wing arrangement.

Head Resistance and Thrust Efficiency. (H. B. Helmbold, Z.F.M., Vol. 22, No. 11, June 15th, 1931, pp. 323-325.) (5.102/22006 Germany.)

Exact measurement of resistance and thrust efficiency at full scale requires measurement by thrust and torque meters. As these were not available, full scale flight performances were reduced in accordance with standard aerodynamic relations and compared with model tests. The types tested were Junkers W.33 and A.35, with Junkers L.5 engines. Rumpler C.VI and Heinkel H.D.22 with B.M.W.IV engines.

Aeroplane Design and Tactical Requirements. (Major T. N. Joyce, Air Services, Vol. 16, No. 7, July, 1931, pp. 37-40.) (5.15/22007 U.S.A.)

The author, who is a test pilot of military aircraft, discusses the relation of design to tactical requirements in a comprehensive manner. Particular attention is given to the field of view. The need of specialised machines in addition to general service machines is maintained.

Relations of Speed, Cost and Range to Aircraft Resistance. (A. Koyemann, Z.F.M., Vol. 22, No. 11, June 15th, 1931, pp. 329-332.) (5.14/22008 Germany.)

Approximate expressions are developed, depending largely on empirical factors. The costs of passenger traffic are worked out numerically in pfennigs per passenger per kilometre and are roughly equal to the figures for railway transport, while the air speed is from six to nine times as great.

Air Races and Design. (W. Wait, Aero Digest, Vol. 18, No. 5, May, 1931, pp. 46-48.) (5.15/22009 U.S.A.)

The author maintains that progress in design comes more rapidly from racing than from mere step by step development, and collects a large number of figures to illustrate his point. He attributes the present backwardness of American military design to their falling out of the Schneider Cup contests.

Method of Least Work in Stressing Aeroplanes. (F. G. Evans, J. Roy. Aer. Soc., Vol. 35, No. 247, July, 1931, pp. 642-644.) (5.15/22010 Great Britain.)

The elementary theory is developed and methods of application are outlined. Correction for secondary stresses is indicated.

Mechanical Testing in Aircraft Construction. (I. J. Gerard, J. Roy. Aer. Soc., Vol. 35, No. 247, July, 1931, pp. 579-608.) (5.15/22011 Great Britain.)

The methods of testing imposed by the requirements of the A.I.D. are discussed and illustrated by sketches of test pieces. Photographs exhibit spars, pipes, frames and hulls under load factor tests. A brief note on testing engine components is added.

Interaction of the Spars in Cantilever Wings. (E. Abit, L'Aeronautique, No. 144, May, 1931, p. 172.) (5.252/22012 France.)

Previous work, mainly by German writers, is reviewed and certain criticisms are offered. The author appeals to test results which indicate important reactions between the main spars under ordinary conditions of loading, and devises a new scheme of calculation taking this factor into account. The details of computation are omitted, but numerical results are embodied in curves and numerical examples. Finally it is claimed for the method that the best utilisation of material is rendered possible.

Gerin Variable Surface Wing. (L'Aeronautique, No. 144, May, 1931, pp. 188-190.) (5.254/22013 France.)

Photographs and diagrams illustrate the construction and operation of the Gerin variable surface wing. See Abstr. No. 19/21314.

Investigation of Aeroplanes with Variable Wings. (W. Schmeidler, Z.F.M., Vol. 22, No. 11, June 15th, 1931, pp. 325-329.) (5.254/22014 Germany.)

The distribution of circulation along the wing is discussed mathematically, and results, given numerically and graphically, are in fair agreement with measured results.

Starting and Landing of Aircraft without Undercarriages. (A. Pröll, Z.F.M., Vol. 22, No. 9, May 15th, 1931, pp. 255-258.) (5.50/22015 Germany.)

The saving in weight and resistance obtained by dispensing with a landing carriage is so great that the possibilities of starting from a trolley or detachable carriage and landing without a carriage are considered worthy of detailed discussion. The principal difficulty in landing is the need for maintaining clearance between the airscrew and the ground.

Tests with Aeroplane Brakes. (Conclusion of 222nd D.V.L. Report, Z.F.M., Vol. 22, No. 11, June 15th, 1931, pp. 338-344.) (5.58/22016 Germany.)

In continuation of the first part, the effect of braking landing wheels is discussed. The moment about the centre of gravity on different types of soil is shown graphically against time of taxying. The wearing properties of various facing materials are tabulated. Photographs show the effects of wear and the mounting of the load indicator. Comparisons are made with the braking of automobile wheels.

Aeroplane Braking Systems. (R. Waring-Brown, Airc. Eng., Vol. 3, Nos. 28/29, June/July, 1931, pp. 139-140 and 157-161.) (5.58/22017 Great Britain.)

The not inconsiderable difficulties met in applications of braking to aeroplanes are discussed clearly. Elementary formulæ are derived for the braking drag obtainable and the effect in reducing landing run. Various relations between machine weight, drag, attitude, aerodynamical drag, brake drag, wheel position, and wheel loading are given graphically. Variations of coefficients of friction of brake lining with temperature, pressure and rubbing speed are plotted. Servo brakes are discussed and a number of mechanical details are illustrated.

Experiments with Floats. (F. Seewald, Z.F.M., Vol. 22, No. 9, May 15th, 1931, pp. 265-276, 219th D.V.L. Report, Pt. 1.) (5.51/22018 Germany.)

The paper was read at the D.V.L. meeting 18. 7. 30. The author investigates the resistance of floats during the start and compares them with English model results. It is pointed out that the usual model experiment takes place within the region of low Reynolds numbers in which the surface flow is partly laminar and partly turbulent, and the total resistance falls short of that observed with high Reynolds numbers. The coefficients of moment of resistance of a float are exhibited graphically for constant loading and speed as functions of the trim angle for three different velocities, all lower than the starting velocity.

The moments of the wing lift, airscrew thrust and control surface forces about the centre of gravity are also shown graphically for different positions of the elevator.

The inferences to be drawn from Pabst's paper on landing shocks (see Abstract No. 14/12779) are discussed at length.

Seaplanes—Flotation Gear. (V. W. Harshman, Air Services, Vol. 16, No. 6, June, 1931, pp. 30-32.) (5.51/22019 U.S.A.)

A pilot after a forced descent at sea remained afloat in a rubber boat for five days. His log is reproduced and is illustrated by a photograph of the boat, inflated by the use of compressed carbon dioxide.

See Abstract No. 19/21306.

Variable Pitch Airscrew. (Autom. Ind., Vol. 64, No. 24, June 13th, 1931.) (5.658/22020 U.S.A.)

The blades of a Curtiss variable pitch airscrew are rotated to the required setting by a small electric motor. The drive is geared down heavily so that the required motor torque is small, and the storage battery for lighting supplies the power. The centrifugal force on the blade roots offered the most serious mechanical problem.

No design details are given.

Instruments

Metering of Large Volumes of Air. (H. S. Bean, M. E. Benesh, and E. Buckingham, Bur. Stan. J. Res., Vol. 7, No. 1, July, 1931, pp. 93-145.) (6.22/22021 U.S.A.)

From authors' abstract:—

Experiments were carried out on the flow of air from a large gas-holder through a rotary displacement gas meter of 200,000 c.f. per hour rated capacity, a 24 by 12 inch Venturi meter, and various thin-plate orifices installed in a smooth 24 in. pipe. The rate of flow was determined from observations on the holder, which was flooded by a stream of water, to control the temperature. The average precision of a determination was better than ±0.5 per cent.

When the values obtained for the discharge coefficients of the orifices were so reduced as to be comparable with the most complete and accurate published values for water, the agreement was very satisfactory.

The Functioning of the Pitot Tube. (L. Escande and M. Teissié Solier, C.R., Vol. 192, No. 19, May 11th, 1931, pp. 1152-1154.) (6.251/22022 France.)

The effect of the position of the lateral orifice was measured at various distances downstream. The coefficient approaches unity very closely. The orifice is moved nearer and nearer the hemispherical head and finally over it the calibrated curve departs altogether from the straight line indicating constant unit value.

The Effect of Working Conditions on a Pitot Tube. (Teissié Solier, C.R., Vol. 192, No. 21, May 26th, 1931, pp. 1306-1309.) (6.251/22023 France.)

The calibration of a pitot tube was carried out in non-turbulent and turbulent flow. The author states that the coefficient remains practically unity under both sets of conditions.

The impulse of a turbulent jet upon a flat plate was also measured, and was found to agree closely with the calculated impulse neglecting the turbulence.

Diaphragm Pressure Elements. (T. Theodorsen, N.A.C.A. Rept. No. 388, July, 1931.) (6.251/22024 U.S.A.)

The calibration of diaphragms of pressure elements shows changes with temperature, and ageing effects with time. The temperature factor was found to arise from difference between the coefficient of expansion of the diaphragms and of the cylindrical wall of the element. Methods for compensation are described. Internal hysteresis was found to be negligible, and lag was found to be due to the friction of the point of the stylus. Weaker springs controlling the stylus gave better results. Other errors were found to be due to bad adjustment. Calibration results are given graphically for useful ranges of temperature and pressure. The elementary elastic theory of deformed membranes is given.

A New Instrument for Accurate Measurement of Air Pressure in Mine Ventilating Shafts. (E. Stach and H. Kirsten, Z. Instrum., Vol. 51, No. 2, February, 1931, pp. 106-107.) (6.252/22025 Germany.)

The Askania statoscope adapted to air pressure measurements in mines indicates pressure differences of the order of .1 mm. of mercury in ventilating shafts.

A Balanced Diaphragm Indicator of Maximum Cylinder Pressure. (J. A. Spanogle and J. H. Collins, N.A.C.A. Tech. Note No. 359.) (6.251/22026 U.S.A.)

The diaphragm is contained in a compact pressure element which brings it within $\frac{1}{8}$ in. of the combustion chamber walls without necessitating water cooling. Contact of the diaphragm with a stop encloses a circuit containing a Neon lamp. In measuring maximum explosion pressures a range of pressures is taken which shows the variation between cycles. The indicator is in use at the Langley Memorial Aeronautical Laboratory, and is considered the most accurate available instrument for indicating maximum cylinder pressures.

Piezo-Electric Measurement of Rotary Acceleration. (Z. Instrum., Vol. 51, No. 5, May, 1931, p. 239.) (6.44/22027 Germany.)

The German Reichsanstalt have constructed a piezo-electric device for measuring engine acceleration. It virtually consists of a small flywheel, acting as an inertia mass and driven by the engine crankshaft through quartz crystals. The arrangement is balanced against centrifugal acceleration.

A New Turn Indicator. (Aire. Eng., Vol. 3, No. 28, June, 1931, p. 146.) (6.52/22028 Great Britain.)

A brief description, illustrated by a sectional sketch, is given of a gyroscopic turn indicator in which some novel mechanical details are applied. The diameter of the casing is given as 4 in., and total weight as 25 ounces. It is stated that test flights have given useful results.

Course Indicator of Pointer Type for Visual Radio Range-Beacon System. (F. W. Dunmore, Bur. Stan. J. Res., Vol. 7, No. 1, July, 1931, pp. 147-170.) (6.55/22029 U.S.A.)

From author's abstract:—

A form of tuned-reed radio range-beacon course indicator is described, called a reed converter, in which the course indications are not given by observing the two reed motions as heretofore, but by means of a zero-centre pointer type indicating instrument. The motion of the two reeds generates small alternating voltages, which, when rectified by oxide rectifiers and passed in opposing polarities through a zero-centre indicating instrument, serve to give course indications by the deflection of the indicating instrument needle in the direction of deviation of the aeroplane from the course.

A New Device for Shock-Proof Mounting of Measuring Instruments. (R. Müller, Z. Instrum., Vol. 51, No. 2, February, 1931, pp. 95-97.) (6.9/22030 Germany.)

The instrument is mounted on a platform which has torsional motion with a free period of about four seconds under spring control and liquid damping.

The device is compact and easily fitted or removed, and enables steady and accurate readings to be made in buildings subject to vibration without providing independent foundations.

Stability and Control

Longitudinal Stability of the Canard Type. (Z.F.M., Vol. 22, No. 11, June 15th, 1931, p. 333.) (7.25/22031 Germany.)

A complete enumeration is given in tabular form of possible arrangements of wings and control surfaces for longitudinal stability.

Focke-Wulf Canard. (H. Focke, L'Aeron., No. 144, May, 1931, pp. 165-171.) (7.25/22032 France.)

The designer, Prof. H. Focke, gives a brief specification of the Focke-Wulf and an elementary examination of the stability problems, which take a somewhat special form. A more technical note follows by W. Margoulis, on the relative stability of the normal aeroplane and of the Canard type, and formally demonstrates certain disabilities imposed on the latter by the relatively high loading of the stabilising surface on account of its forward position.

Method of Flight Measurement of Spins. (H. A. Soulé and N. F. Scudder, N.A.C.A. Rept. No. 377, May, 1931.) (7.62/22033 U.S.A.)

The test aeroplane was equipped with:

(*a*) three single component turn-meters affording three angular velocities about the main axis,

(*b*) a three-component accelerometer at the c.g. of the aeroplane affording acceleration along the aeroplane axes,

(*c*) a recording statoscope and special altimeter; the former was found to agree with the change of height measured by camera-obscura, and to be more convenient in operation. The recording altimeter was found to be inaccurate.

Two training aeroplanes were used, and the specification of their main characteristics is given with dimensioned sketches and photographs of the equipment. A list of symbols and formulæ used is given, and tables of measured and computed quantities. It is considered that the accuracy is sufficient to determine changes in design.

Eleven references are given.

Thermodynamics, Engines, Etc.

Current German Aircraft Engine Research. (Autom. Ind., Vol. 65, No. 5, August 1st, 1931, p. 155.) (8.1/22034 U.S.A.)

According to D.V.L. reports, the direct injection of light fuels in two-stroke spark-ignition engines offers certain advantages. An experimental aircraft engine is being constructed.

The Theory of the Combustion Chamber. (Jacoby, Autom. Zeit., Vol. 34, No. 14, May 20th, 1931, pp. 332-335.) (8.10/22035 Germany.)

To raise the compression ratio of an engine to its maximum possible value the compression space should be flat to give intensive cooling to the portion of the gas which burns last. The sparking-plug should be placed as near as possible to the exhaust valve; so that the last portion of gas to burn may not be compressed against this hot spot.

Equation of State of Propellant Gases. (A. D. Crow and W. E. Grimshaw, Trans. Roy. Soc., Lond., No. A.682, February 13th, 1931, pp. 39-73.) (8.10/22036 Great Britain.)

From authors' summary:—

To formulate an expression for the equation of state of the gases resulting from explosions of propellant of known chemical composition it has been found necessary:—

1. To evolve adequate methods for the measurement of pressure in the conditions encountered;
2. To investigate in detail the nature of, and to establish an expression for, the energy losses arising from the cooling of the hot gases at the wall surface of the vessel, together with the work absorbed in wall stress;
3. To re-formulate in the light of the latest available data expressions for assessing the molecular heats at constant volume of the various constituents of the gas complex, for temperatures ranging from 2700°K. to 4200°K.;
4. To re-calculate the equilibrium constant for the water-gas reaction over the range of temperatures involved;
5. To utilise the experimental evidence available for determining the effective volume occupied by the gas molecules.

Agreement to within 2 per cent. has been established for the ballistic force of the propellant, between the values derived from direct experiment and those obtained from thermodynamical considerations.

The Mechanics of Fire. (H. E. Armstrong, Jrnl. Soc. Chem. Ind., July, 1931, Jubilee No., p. 179.) (8.1/22037 Great Britain.)

This lecture by Prof. Armstrong in 1905 contains a remarkable forecast of many recent theories of combustion, including autoxidation, peroxidation and hydroxydation. Twenty-one references from 1867 to 1904 are given.

Combustion Process in the Injection Engine Using Pre-Combustion Chamber. (E. Schmidt, Z.V.D.I., Vol. 75, No. 19, May 9th, 1931, pp. 585-591.) (8.10/22038 Germany.)

The temperature distribution in an engine cylinder of a 10 h.p. 450 r.p.m. two-stroke crank scavenger by Deutz was measured by inserting thermo-couples and registering on a string galvanometer. The couples were of the platinum-rhodium type and ranged in thickness from 0.03 to 0.05 mm. The records obtained show the distribution of the flame in the pre-combustion chamber and in the main cylinder. The scavenging is indicated by discontinuities in the temperature curves. The absolute temperatures recorded may be affected by temperature

lag, but the motion of the scavenger air, as traced out by the couples, throws light on the operation of the cycle.

Engine Performance at High Compression Ratios. (Autom. Ind., Vol. 65, No. 5, August 1st, 1931, p. 175.) (8.1/22039 U.S.A.)

Experiments carried out on a N.A.C.A. universal test engine by the Engineering Research Department of the University of Michigan show that equal power outputs can be obtained from a high compression engine with equal intensity of detonation using fuels of very different antiknock properties, provided the spark can be adjusted over a wide enough range.

Superchargers and Supercharging. (O. W. Schey, Airc. Eng., Vol. 3, No. 28, June, 1931, pp. 134-138.) (8.235/22040 U.S.A.)

The data obtained in N.A.C.A. experiments are collected in a convenient form for reference and use by designers. Different types of superchargers are discussed and the principles illustrated diagrammatically. Numerous curves are reproduced showing comparative performance with various types of supercharger. The selection of a suitable type and capacity of supercharger is discussed and numerical values are assigned to the gain in maximum altitude, rate of climb, and speed.

Some Factors Influencing the Sizes of Crankshafts for Double-Acting Diesel Engines. (S. F. Dorey, J. N.E. Coast Inst. Eng., Vol. 47, No. 6, June-July, 1931, pp. 229-316.) (8.25/22041 Great Britain.)

Author's summary:—

The diameter of a shaft subjected to combined bending and torsion is calculated from the equivalent bending or twisting moment to a constant working stress, irrespective of the type of engine. In the paper a different working stress is allowed for each type of engine according to the ratio of twisting moment to bending moment, and also to the fluctuations of shear stress at the most severely stressed crank. The diameter of a crankshaft subjected to large fluctuations of torque is allowed a lower working stress than a shaft with the same maximum torque subjected to small fluctuations.

The method indicated in the first part of the paper shows how the strength of crankshafts can be ascertained from a knowledge of the fatigue properties of materials at various ranges of stress, and has been used to determine suitable diameters of crankshafts for double-acting Diesel engines, based on previous experience with steam reciprocating engines and single acting Diesel engines. The results indicate that for engines having a small number of cylinders the working stress permissible should be appreciably less than for engines having a large number of cylinders for the same factor of safety.

For two-stroke-cycle double-acting engines having a large number of cylinders the maximum working torsional stress due to combined twisting and bending suggested is somewhat in excess of that usually adopted in existing steam engines and single-acting Diesel engines, viz., 7,000-7,500 lbs. per sq. in., as against 5,600 lbs. per sq. in., and as a result some comments have been made relative to the shrinkage grip for built shafts.

Compression-Ignition Engines. (H. T. Tizard, Airc. Eng., Vol. 3, No. 30, August, 1931, pp. 185-186.) (8.25/22042 Great Britain.)

The author discusses the physical principles of combustion and power production in petrol engines, where the combustion is regarded as taking place with constant volume, and in compression ignition engines, in which combustion in the limit takes place at constant pressure but in practice between conditions of constant pressure and constant volume.

Using recent figures for the specific heats of the gaseous products of combustion and for the dissociation of water and carbon dioxide, ideal efficiences are calculated and shown graphically as functions of maximum pressure. A discussion of the relations between these quantities, taking into account the strength and radiating surface of the two types, indicates that the necessary compromise limits maximum pressure in a very definite way. The greater radiating surface of the compression ignition engine leads to the suggestion that the two-stroke cycle with initial pressures above atmospheric renders possible a design which will compete successfully with the petrol engine.

Heavy-Oil Engine Research in U.S.A. (W. F. Joachim, Z.V.D.I., Vol. 75, No. 3, January 17th, 1931, pp. 69-76.) (8.25/22043 Germany.)

Recent research work in America on Diesel engines is viewed critically with numerous extracts from the test results, illustrations of test apparatus, graphical charts of temperatures and pressures, delivery of fuel from nozzle, indicator diagrams, etc. A table is reproduced from the experience of Busch-Sulzer Bros. Diesel Engine Co., showing replacements in over six years for 100 Diesel engines kept under observation.

Numerous references are given to German work in the same direction.

The Michel Diesel Engine. (Autom. Zeit., Vol. 34, No. 11, May 20th, 1931, pp. 258-263.) (8.25/22044 Germany.)

In the Michel opposed cylinder engine three pistons move simultaneously with reference to a star-shaped combustion chamber. In a previous design the three pistons operated a hollow drum by roller and cam-track motion. The power output is taken from the rotating drum, which acts as a reduction gear. This arrangement is only suitable for ship propulsion. The article describes a later design in which three pistons of one unit operate on three separate crank-shafts, which are inter-geared by means of a triangular link plate. In this way an arrangement suitable for lorry work is obtained. Performance figures on an experimental unit show that reasonable m.e.p. can be held at 2,000 r.p.m. Work is in progress to adapt the design to aircraft purposes.

The Junkers Heavy-Oil Engine "Jumo IV." (Die Luftwacht, No. 6, June, 1931, pp. 284-285.) (8.26/22045 Germany.)

During a 50-hour type-test in five runs of 10 hours each under D.V.L. regulations, an average of 645 h.p. was developed at 1,710 r.p.m. The fuel consumption was .37 lbs. of gas-oil per brake-horsepower-hour. The lubricating oil consumption was relatively high at .033 lbs. per brake-horsepower-hour. During the run no critical periods were found for torsional vibration over the normal range between 1,300 and 1,800 r.p.m. On dismantling after the test no undue wear was discovered in any part. The engine is now undergoing flight test in a freight carrier.

Direct Fuel Injection for Four-Stroke Engines Using Electric Ignition. (Autom. Zeit., Vol. 34, No. 11, April 20th, 1931, p. 265.) (8.25/22046 Germany.)

A review is given of experiments carried out at the Massachusetts Institute of Technology. The fuel was injected by a Bosch pump, the duration of injection varying between 19° and 35° crank angle, either into the induction manifold or directly into the cylinder. The maximum possible compression ratio is not changed appreciably, but fuel injection increases the volumetric efficiency, and hence the power about 10 per cent. over that obtained with a carburettor.

Isovolt Electrodes for Sparking Plugs. (Autom. Zeit., Vol. 33, No. 25, September 10th, 1930, pp. 611-612.) (8.283/22047 Germany.)

It is claimed that a new alloy for sparking-plug electrodes, introduced by the A.C. company as Isovolt, does not burn away as rapidly as steel electrodes and requires a smaller sparking potential by its effective emission of electrons.

Stresses in Combustion Engines. (Autom. Zeit., Vol. 33, No. 25, September 10th, 1930, pp. 604-606.) (8.32/22048 Germany.)

Piston side pressure over a full working cycle is increased by off-setting the piston and displacing the gudgeon pin from the axis of symmetry. Cylinder wear is mainly due to piston ring friction and should be reduced by using rings of wider section.

The Balancing of Crankshafts. (W. Späth, Autom. Zeit., Vol. 33, No. 28, October 10th, 1930, pp. 671-675.) (8.34/22049 Germany.)

In slow speed balancing machines the crankshaft is usually supported by two bearings at the ends. At high speeds a shaft with unbalanced webs will deflect in the middle. If this is corrected by the machine considerable out-of-balance forces appear at low speeds. In the engine the shaft is usually supported by more than two bearings and the resultant deflection will depend upon the play in the bearings. The author describes a method of balancing with three bearings. Satisfactory results are obtained only if each throw of the crankshaft is separately balanced taking into account the effective mass of the connecting rod.

New Vibration Recording Apparatus. (W. Späth, Z.V.D.I., Vol. 75, No. 3, January 17th, 1931, pp. 83-85.) (6.104/22050 Germany.)

A brief note is given on developments of vibration recording instruments and their application.

Borg and Beck Coupling with New Oscillation Damper. (Autom. Zeit., Vol. 34, No. 2, January 20th, 1931, p. 32.) (8.765/22051 Germany.)

The coupling keeps the rubber cushion drive always under compression, which gives increased life of the material.

Cooling

Cooling System for Lubricating Oils. (British Pat. 339650, September 10th, 1929. T. R. Cave-Browne-Cave.) (8.4/22052 Great Britain.)

A cooling coil is mounted inside the oil tank and connected to an outside condenser. Carbon tetrachloride is suggested as a suitable cooling medium.

International Aero Exhibition, December, 1930, Paris. (Z.V.D.I., Vol. 75, No. 16, April 18th, 1931, pp. 473-480.) (8.40/22053 Germany.)

The majority of engines exhibited were air-cooled from 150 h.p. to 300 h.p. Water-cooled engines were mostly from 500 h.p. to 600 h.p. For many fighting purposes 300 h.p. is considered sufficient by the French authorities. The head resistance depends on the required cooling which takes place primarily in the boundary layer located on the fins of the air-cooled cylinder or on the cooling tubes of the radiator. If attempts are made to reduce the resistance below a certain amount the engine overheats. There does not appear to be much difference between the head resistance of the two types, nor does the air-cooled in-line differ appreciably from the radial engine. In the former case the guiding of the air from one cylinder to another requires cowling and guide vanes. The radial engine requires no guiding of the air, and, in spite of bigger cross-section, has equally low resistance, especially if the flow is controlled by ring cowling.

Endurance of Air-Cooled Engines. (Autom. Tech. Zeit., Vol. 34, No. 8, March 20th, 1931, pp. 188-189.) (8.422/22054 Germany.)

Statistics of endurance are collected from various foreign sources for the Rhone-Jupiter, Bristol-Jupiter, Cirrus and Wright-Whirlwind engines. The wide differences in the figures suggest that there is still a considerable factor of ignorance of engine performance.

Cooling of Engines. (A. Nutt, Airc. Eng., Vol. 3, No. 29, July, 1931, pp. 172-173. Nat. Aeronautic Meeting S.A.E., April, 1931, Detroit.) (8.44/22055 U.S.A.)

Cooling is discussed largely in relation to improved power output of the engine concerned by the use of a high temperature cooling medium. Ethylene-glycol and di-ethylene-glycol have been found satisfactory. Seven sets of curves show the experimental results obtained.

Patent Wing Radiator. (French Patent No. 701546, December 3rd, 1929, Nieuport-Astra. L'Aerophile, Vol. 39, No. 6, June 15th, 1931, p. 186.) (8.464/22056 France.)

The radiator consists of two metal sheets following the wing profile, between which the cooling liquid circulates. The radiator sections are attached at their edges and at intermediate points at which they are also inter-connected to form a closed circuit with the engine jackets.

Lubricants and Fuels

Carbonisation of Lubricating Oil. (H. N. Bassett, Chem. and Ind., Vol. 50, No. 26, June 26th, 1931, pp. 527-529.) (8.540/22057 Great Britain.)

The tendency of a lubricating oil to produce carbonaceous or gummy deposits in an engine is an important characteristic. The various laboratory methods of investigation may be divided into three groups depending on (*a*) volatility, (*b*) coking, and (*c*) oxidation. None of these reproduce engine conditions satisfactorily, and the rating of lubricating oil can be determined only by engine tests.

The Lubricating Properties of Mineral, Vegetable and Fatty Oil. (A. S. T. Thomson and P. S. Caldwell, J. Roy. Tech. Coll. (Glasgow), 2, 490-502 (1931). Chem. Absts., Vol. 25, No. 11, June 10th, 1931, p. 2842.) (8.54/22058 Great Britain.)

Tests were carried out on pure mineral oil, rape oil, castor oil, sperm oil and blended oils. A new apparatus is described for investigating the lubrication of cylinder bearings of the ring oiler type. The " oiliness " factor of mineral oils is increased by the addition of small quantities of fatty oils.

Lubricating Quality of Oils. (Z. Instrum., Vol. 51, No. 5, May, 1931, p. 237.) (8.54/22059 Germany.)

Indications of the boundary lubrication conditions are sought from the electrical rectifying property of the oil film. The displacement of the shaft in the bearing is measured. The beginning of rectification and minimum friction occur near the same speed and are thus associated with the thickness of the film.

Failure of Lubrication Under High Tooth Loading. (W. C. Bauer, Autom. Ind., Vol. 64, No. 24, June 13th, 1931, pp. 910-912.) (8.540/22060 U.S.A.)

The Standard Oil Development Co. have long held that the quality of an oil can be assessed only by tests to destruction, particularly for heavy-duty lubricants in the hypoid gear.

In 43 runs on a low-speed heavy duty chassis, 18 out of 29 lubricants failed.

The Use of Alcohol for Motor Fuels. (Karel Loskot, Chem. Ind., 25, 37-44, 62-7 (1931). Chem. Absts., Vol. 25, No. 10, May 20th, 1931, p. 2541.) (8.606/22061 U.S.A.)

Good results have been obtained from a fuel mixture of composition:—

Petrol	37·5 per cent.
Ether alcohol	37·5 ,, ,,
Benzine	25 ,, ,,

The alcohol must be anhydrous.

A fuel mixture, 60 per cent. petrol, 40 per cent. alcohol (99 per cent. anhydrous) has also given good results.

Mixing Alcohol and Motor Spirit. (K. R. Dietrich, Autom. Zeit., Vol. 33, No. 36, December 31st, 1930, pp. 873-877.) (8.606/22062 Germany.)

Five sectional diagrams are given of tanks equipped with apparatus for mixing alcohol and motor spirit to meet German legal obligations in this respect.

Fuel Quality. (F. R. Banks, Airc. Eng., Vol. 3, No. 29, July, 1931, pp. 168-170.) (8.64/22063 Great Britain.)

The author discusses fuel quality largely from the U.S. point of view, but also with reference to British Air Ministry specifications. Great importance is ascribed to suitable blending of fuels, and the remark is made that engine-makers in the U.S. can correlate the distribution of engine failures with local types of blended fuels used. The two most important points are wear and tear from high knock temperatures. The whole discussion is illustrated by references to particular fuels actually in use, and to selected specifications for valves and valve seatings.

Memorandum on Petrol-Alcohol-Benzole Mixtures. (C. Baron, C. Boulanger, R. Le Grain, C.R., Vol. 192, No. 22, June 1st, 1931, pp. 1383-1385.) (8.64/22064 France.)

Freezing experiments were carried out on a series of alcohol-petrol-benzole mixtures, as a result of which fuel consisting of 70 per cent. petrol (aviation), 20 per cent. benzole, and 10 per cent. alcohol is recommended as fulfilling best normal flying conditions.

Motor Fuel. (German Patent No. 520011, June 13th, 1926, E. G. E. Meyer.) (8.64/22065 Germany.)

The fuel consists of a mixture of a hydrocarbon distillate containing 1-5 per cent. of ether and a small quantity of a volatile base such as ammonia or methylamine.

Latent Heat of Evaporation in Fuels. (Wawrziniok, Autom. Tech. Zeit., Vol. 33, No. 26, September 20th, 1930, pp. 644-646, and No. 31, November 10th, 1930, pp. 764-766.) (8.640/22066 Germany.)

A method is described for determining the latent heat of evaporation of various fuels. The results for pure benzol, pure alcohol, and four intermediate mixtures of the two are given graphically against the percentage volume evaporated. Tables and graphical representations of several types of fuels and fuel mixtures are also given. On comparing the experimental results with calculated values it is concluded that the latter have sufficient accuracy for practical purposes.

Fuels. (Chem. and Ind., Vol. 50, No. 27, July 3rd, 1931.) (8.640/22067 Great Britain.)

In a symposium of fuel utilisation are papers on Gaseous Combustion in Industry, R. V. Wheeler, p. 550; Refining Progress in Petroleum Development, A. E. Dunstan, p. 557; and Low Temperature Tar, T. F. Hurley and M. A. Matthews, p. 584.

Fuel. (German Patent No. 520010, March 26th, 1929, A. J. Duncamp.) (8.64/22068 Germany.)

Premature combustion of the fuel or lubricant is prevented by the addition of mercury cyanide. The salt may be dissolved in a mixture of alcohol and ether and added to the fuel. Alternatively the salt may be dissolved in glycerol and added to the lubricating oil. The proportions of salt required range from .1 to 1 gram of cyanide to 10 litres of fuel.

High Temperature Knock Testing. (G. Edgar, Airc. Eng., Vol. 3, No. 29, July, 1931, p. 171. National Aeronautic Meeting S.A.E., April 1931, Detroit.) (8.645/22069 U.S.A.)

Curves and tables are given showing tetra-ethyl lead content against octane numbers for various blended fuels, so as to give satisfactory matching.

Photographic Determination of the Detonation Resistance of Petrols. (M. Aubert and R. Duchêne, C.R., Vol. 192, No. 25, June 22nd, 1931, pp. 1633-1635.) (8.645/22070 France.)

Flame photographs of an explosion in a bomb show two distinct features. The first portion of the flame, initiated at the sparking-plug, travels at relatively small speed and is not very actinic till it reaches the walls of the confining vessel, where it is reflected; and the reflected flame is generally much more actinic. The intensity of detonation depends on the relative intensity of these two phenomena. The authors propose a method of knock-rating fuels. A series of flame photographs with gradually increasing bomb temperatures shows a diminution in intensity of the flame travelling outwards from the plug, and an increase in the intensity of the reflected flame. The temperature at which detonation first takes place is marked by the failure of the first flame to register, while the detonation is recorded at some period subsequent to the passage of the spark preceded by the apparent blank on the negative. This temperature can be recorded with precision and is defined as the detonating characteristic of the fuel. By adding a dope, but keeping the temperatures the same, detonation is reduced and the first part of the flame reappears. The authors state that the classification of fuels obtained by them has been found to hold in actual engine tests.

Anti-Knock Motor Fuel. (British Patent No. 339637, July 10th, 1929, I. G. Farbenind.) (8.645/22071 Great Britain.)

The dope consists of a mixture of an amine such as dimethyl aniline and iron carbonyl.

The Behaviour of Anti-Knocks. (A. Egerton and L. M. Pidgeon, Nature, 127, p. 591 (1931). Chem. Absts., Vol. 25, No. 12, June 20th, 1931, p. 3156.) (8.645/22072 Germany.)

An experiment shows that lead tetra-ethyl must become oxidised before it acts as an effective anti-knock.

Anti-Aircraft Guns, Etc.

Aircraft Defence. (U.S. Nav. Inst. Proc., Vol. 56, No. 333, November, 1930, pp. 1034-1036.) (9.11/22073 U.S.A.)

In a descriptive article, quoted from the daily press, a number of technical details are given of anti-aircraft equipment and training.

Anti-Aircraft Gun Control. (R. H. Ward, Army Ordnance, Vol. XI., No. 66, May-June, 1931, pp. 452-457.) (9.61/22074 U.S.A.)

An elementary theory of range determination is illustrated by diagrams. The scheme of connections between a Sperry anti-aircraft gun control system mounted

on a truck and a battery of aircraft guns is reproduced. A photograph shows the battery and control in action. The principal requirements of anti-aircraft control are summarised.

Transportation of Artillery by Air. (L. B. Ely, Aero Digest, Vol. 18, No. 6, June, 1931, pp. 38-39.) (9.61/22075 U.S.A.)

An operational account is given of the transport by aeroplane of a three-inch four-gun howitzer battery, with crews.

Materials

Cast Iron Containing Molybdenum. (Autom. Ind., Vol. 65, No. 5, August 1st, 1931, pp. 168-171.) (10.1/22076 U.S.A.)

The physical properties of the whole range of cast irons are improved by the addition of .4 to 1.4 per cent. of molybdenum. The tensile strength, fatigue limit, Brinell hardness, and resistance to wear are improved, without loss of machining qualities.

Some New Anti-Corrosion Protective Coverings for Iron and Steel. (H. Reinniger, Autom. Zeit., Vol. 34, No. 2, January 20th, 1931, pp. 27-29.) (10.125/22077 Germany.)

Three methods are described. One consists of spraying with aluminium; the other two are chemical in the sense that an iron oxide or iron phosphate deposit is produced on the steel. The latter has received a fairly wide application.

" Ni-resist," a New Cylinder Material. (Mahle, Autom. Zeit., Vol. 34, No. 11, April 20th, 1931, p. 257.) (10.101/22078 Germany.)

The International Nickel Co. has introduced a new iron nickel alloy for which great advantages are claimed as a cylinder liner material. Its thermal expansion is stated to be as high as .000019, compared with .000012 for cast iron, and with a light alloy piston of similar thermal expansion close working fits can be maintained at all temperatures. According to tests carried out by the German firm producing Electron the thermal expansion of the material does not generally exceed .000016. The higher the thermal expansion the lower is the hardness, and the heat conductivity is only about one quarter of that of cast iron.

Nickel-Iron Alloys. (F. E. J. Ockenden, J. Sci. Insts., Vol. 8, No. 4, April, 1931, pp. 113-117.) (10.210/22079 Great Britain.)

The properties are described of a new group of nickel-iron alloys which give extremely high permeability and low core losses. Various applications to the design of instruments are suggested.

Influence of Acetylene Pressure on the Quality of Welded Joints. (K. Gabler, Z.V.D.I., Vol. 75, No. 3, January 17th, 1931, pp. 77-81.) (10.140/22080 Germany.)

The effect of pressure on the delivery of the jet, the mixing with air, and the composition of the burnt gases is discussed in relation to their effect on extension and tensile strength of welds. The results are exhibited graphically in numerous diagrams.

Investigations on the Thermal Expansion of Aluminium Alloys. (H. Sieglerschmidt. Z. Metallk., Vol. 23, No. 1, January, 1931, p. 26-30.) (10.231/22081 Germany.)

Comparative experiments were carried out on duralumin, lautal (5 per cent. copper), and on pure and commercial aluminium. The coefficient of expansion

depends on the temperature and state of the sample, especially after cold working. The coefficient increases with temperature, with stationary points indicating the crystallisation of the metal. The opinion is expressed that this new method of studying thermal expansion will throw light on the theory of alloy structure.

The Process of Ageing in Various Aluminium Sand-Cast Alloys. (W. Saran, Z. Metallk., Vol. 23, No. 1, January, 1931, pp. 32-33.) (10.231/22082 Germany.)

Two types of alloys were tested, one requiring heat treatment after casting, the other ageing at room temperature.

The ageing process was completed in 12 days, and the heat treatment in 20 days. Brinell tests on the heat treated alloys showed day to day variations in the hardness, possibly due to periodic changes in the room temperature.

Magnesium. (J. A. Gann, S.A.E., Vol. 28, No. 6, June, 1931, pp. 653-668.) (10.232/22083 U.S.A.)

The author describes foundry practice in the production of magnesium and magnesium-alloy castings, and the effect of heat-treatment and fabrication processes on the microstructure and physical properties.

Applications of magnesium alloys to aircraft and automobiles are outlined.

The importance of magnesium as a structural metal is now being recognised, as the factors that have retarded its development and restricted its use are overcome.

Dural Plate Riveted Joints, Corrosion Tests. (P. Brenner, Z.F.M., Vol. 22, No. 11, June 15th, 1931, pp. 344-346.) (10.262/22084 Germany.)

The loss of strength and extensibility after 80 days' exposure to corrosion by salt water spray with 20 per cent. Na.Cl. solution is tabulated. Seven photographs show the surface appearance and the sectional view of the rivets. The worst effects are avoided by counter-sinking the heads of the rivets in the riveted plates.

Wind Tunnels, Testing Gear, Etc.

An American Wind-Channel. (R. J. Fairbanks, Airc. Eng., Vol. 3, No. 28, June, 1931, pp. 147-148.) (11.1/22085 U.S.A.)

A brief account is given of the American 8-foot wind tunnel at the University of Michigan. A number of technical data are given, and three photographs illustrate the equipment of tunnel, test chamber and workshop.

Full-Size Wind-Channel. (Aviation, Vol. 30, No. 7, July, 1931, p. 405.) (11.1/22086 U.S.A.)

A descriptive account is given of the new N.A.C.A. 60′ × 30′ wind channel. It is of the free-jet type and is equipped with two 34′ diameter airscrews absorbing 4,000 h.p. each and producing a flow speed of over 50 m/s. Two photographs show the driving airscrews and a full size biplane mounted on the balance.

Modern Windmills. (Inst. World, Vol. 4, No. 39, July, 1931, p. 56.) (11.12/22087 Great Britain.)

A quotation from a catalogue states that the substitution of two blades with stream-line profiles reduces the thrust and the corresponding stresses on the tower to an appreciable extent.

Method for Measuring Turbulence. (C. F. Taylor, N.A.C.A. Tech. Note No. 380, June, 1931.) (11.16/22088 U.S.A.)

The conductivity of a hot wire under ordinary wind channel conditions and with artificially increased turbulence is plotted for inclinations to the stream

between 0° and 90°. There is in every case a marked maximum near 90°. With artificial turbulence the curve remains almost the same up to about 80°, where it branches off and develops a lower maximum.

It is not considered that the phenomenon is completely analysed, but beginning has been made in setting up some sort of quantitative standard of turbulence.

Experimental Towing Basin. (Aviation, Vol. 30, No. 7, July, 1931, p. 406.) (11.35/22089 U.S.A.)

The new N.A.C.A. towing basin is 2,040 feet long by 24 feet wide by 12 feet deep at the centre line. Some details are given of the travelling carriage and equipment. Speeds up to 50 m.p.h. are obtained.

Two photographs are reproduced.

Sound-Proof Aero-Engine Test Bench at Travemunde. (H. Weidinger, Z.V.D.I., Vol. 75, No. 16, April 18th, 1931, pp. 501-502.) (11.55/22090 Germany.)

The engine is mounted on a Seppeler reaction torque bench, the power being absorbed by an airscrew. The air stream is drawn through a 50 ft. channel with guide vanes, passes over the engine, and is propelled into the open through a second channel. The torque bench is insulated from the air stream by a wall with windows and control apertures. The building has heavy concrete double walls lagged with sound-absorbing material. An air speed of 100 ft. per second can be maintained and is sufficient for the cooling of large air-cooled radial engines.

The sound insulation is sufficiently complete to make it possible to erect the plant in a residential neighbourhood without noise nuisance.

Lubricants and Bearings Test Machine. (E. S. Glauch, Iron Steel Eng., 8, p. 129 (1931). Chem. Absts., Vol. 25, p. 2842, No. 11, June 10th, 1931.) (11.55/22091 U.S.A.)

A new test machine is described capable of taking loads up to 40,000 lbs. The machine is suitable for testing lubricants or bearing materials.

Aerostats, Etc.

Graf Zeppelin Flights, 1930. (Luftwacht, No. 5, May, 1931, p. 203.) (12.10/22092 Germany.)

From April to December, 1930, the airship Graf Zeppelin made 109 flights. The total flying time was 1,273 hours. The average speed was about 60 miles an hour. The number of passengers carried, including crew, was over 7,000.

The Airship "Akron." (B. Jones, Air Services, Vol. 16, No. 6, June, 1931, pp. 21-24.) (12.10/22093 U.S.A.)

A descriptive account of the airship is given, illustrated by two photographs, one of the internal girder construction, the other of the partly-covered airship in its shed.

Helium Output. (U.S. Nav. Inst. Proc., Vol. 56, No. 333, November, 1930, pp. 1046-7.) (12.45/22094 U.S.A.)

Data are given of the output and cost of helium. The Government plant at Amarillo, Texas, produced 1,200,000 c.ft. of helium in the month of May, at a gross operating cost of $11 per thousand feet.

High Altitude Flight of Piccard and Kipfer. (Aero Review, Vol. 6, No. 13, July 1st, 1931, pp. 7-39.) (12.19/22095 Switzerland.)

The balloon had a volume of 14,000 m^3. It was filled with 2,600 m^3 of hydrogen, and the total weight, including fabric, cabin, ballast and crew, amounted

to 2,150 kg. From this a ceiling of approximately 17,000 m. can be calculated, and it appears that an altitude of 16,000 m. was reached. The weather conditions were exceptional. The balloon stayed at an altitude of this order for nearly 17 hours, and finally landed at a distance of less than 120 miles from the starting-point. It was the intention to remain for a short period at the maximum altitude, but the rope operating the gas valve broke and the crew had to await the cooling effect of the evening combined with gas leakage to produce a landing. The article contains a diary of the flight, but no scientific data apart from altitude are given. It appears that ionisation experiments were intended as well as the taking of gas samples. These are promised in a future report.

Wireless

Bibliography of Radio Wave Phenomena, Etc. (Proc. Inst. Rad. Eng., Vol. 19, No. 6, June, 1931, p. 1034.) (13.3/22096 U.S.A.)

A list of 620 references is given, classified under fourteen headings, and appearing in sixty publications. An alphabetical index of authors is added.

Short Radio-Electric Waves. (R. Jonaust and N. Stoyko, C.R., Vol. 192, No. 20, May 18th, 1931, pp. 1207-1209.) (13.30/22097 France.)

In the transmission of wireless waves of 18½m. wave-length from Saigon to Paris, double signals were received with a time interval of about .068 sec. It was inferred that one signal followed the shorter arc of the great circle and the other the longer arc, but the velocity of propagation deduced was slightly less than that of light. On making a correction for the height of the ionised layer the speed of radiation was found to be exactly equal to that of light with the accuracy attainable.

Suppression of Radio-Frequency Harmonics in Transmitters. (J. W. Labus and H. Roder, Proc. Inst. Rad. Engs., Vol. 19, No. 6, June, 1931, pp. 949-962.) (13.31/22098 U.S.A.)

From authors' summary:—

The harmonic components of the antenna current are determined in terms of the corresponding components of the plate current of the power amplifier of transmitters. After investigating the cause of harmonic currents and pointing out the difficulties arising in connection with an exact calculation of the harmonics of the field strength, the discussion is confined to the effect of the circuits inserted between plate and antenna circuit on the suppression of harmonics. Several types of circuits are considered and the current ratios of the harmonic antenna currents with respect to the fundamental are given. The results are tabulated.

Extensions to Theory and Design of Electric Wave-Filters. (O. J. Zobel, Bell Tele. Lab., B.556, April, 1931.) (13.5/22099 U.S.A.)

From author's abstract:—

The problem of terminal wave-filter impedance characteristics is considered, in particular that of obtaining an approximately constant wave-filter impedance in the transmitting bands, which is of importance where the wave-filter is terminated by a constant resistance.

Parts I and II give the derivation and composition of wave-filter structures meeting these requirements. Two allied subjects, respectively, the designs of networks which simulate the impedances of wave-filters, and of loaded lines, are dealt with in Parts III and IV making use of the previous results.

The four appendices contain new reactance and wave-filter frequency theorems, and a chart for determining terminal losses at the junction of a fixed wave-filter transducer with a resistance termination. The chart supplements those previously given in a method of calculating wave-filter transmission losses.

Direct Indication of Small Radio Frequency Differences. (E. and M. Mittelmann, Z.H.F.T., Vol. 37, No. 5, May, 1931, pp. 191-199.) (13.8/22100 Germany.)

A description is given with two photographs and diagrams of the connections of a new type of instrument which indicates directly small frequency differences of the order of 10^{-5}. Where the frequency to be measured approaches the natural frequency of the reactance coil, a sensitivity of the order of 1.4×10^{-6} is obtained.

Automatic Volume Control for Aircraft Radio Receivers. (W. S. Hinman, Jr., Bur. St. J. Res., Vol. 7, No. 1, July, 1931, pp. 37-46.) (13.32/22101 U.S.A.)

From author's abstract:—

An automatic volume-control device is described for use primarily in the reception of visual type radio range-beacon signals, the device being easily applied to existing aircraft radio receiving sets. This device operates on the output voltage of the radio receiver, and is provided with a filter unit to prevent operation of the automatic volume control by signals other than those from the range beacon. The controlling voltage is derived from the output of the radio receiver, part of which is rectified and then applied as negative bias to the radio-frequency amplifier. The automatic volume control maintains a substantially constant output voltage for input voltage variations of the order of 5,000 to 1. A distance indicator, operating in conjunction with the automatic volume-control device, is provided to serve as a gauge of distance from the transmitting station.

Receiver Design for Minimum Fluctuation Noise. (N. P. Case, Proc. Inst. Rad. Eng., Vol. 19, No. 6, June, 1931, pp. 963-970.) (13.32/22102 U.S.A.)

Author's summary:—

The effects of various changes in both tube and circuit conditions are investigated with regard to their influence on the limitations which fluctuation noise sets on the sensitivity of a receiver.

It is concluded that for minimum noise the following conditions should obtain:—The gas pressure in the tube should be less than 10^{-4} mm. of mercury; the antenna-to-grid transfer circuit should be as efficient as possible; the plate-circuit load impedance should be high enough to give a gain of at least five for the first radio frequency tube, apart from the antenna coupling circuit; and the cathode emission should be so high that the tube is always operating under dense space-charge conditions.

Radio Range Beacons, Design and Calibration of Vibrating-Reed Indicators. (G. L. Davies, Bur. St. J. Res., Vol. 7, No. 1, July, 1931, pp. 195-213.) (13.4/22103 U.S.A.)

From author's abstract:—

The paper gives a general treatment of the theory of design of vibrating-reed indicators, which was developed in connection with measurement and design work on the tuned-reed course indicator for the aircraft radio range beacon. The equations and conclusions may be readily adapted to apply to any similar vibrating system.

Aeroplane Antennæ for Radio Range Beacon Reception. (H. Diamond and G. L. Davies, Bur. St. J. Res., Vol. 6, No. 5, May, 1931, pp. 901-916.) (13.4/22104 U.S.A.)

Seven types of antennæ are illustrated by sketches. The mathematical theory of the radiated field and of the reception voltage is discussed by means of vector analysis and the required formulæ derived and expressed in elementary trigonometrical functions. The relative intensity of the received voltage is plotted against the relative distance of aircraft from beacon. The object of the installation is to determine the course. The symmetrical T antenna is recommended as giving the best all-round results.

Directional Transmitting Systems. (E. J. Sterba, Proc. Inst. Rad. Eng., Vol. 19, No. 7, July, 1931, pp. 1184-1215.) (13.6/22105 U.S.A.)

Using Poynting's theorem, the power radiated by an antenna is written down as a double integral over the solid angle. The improvement factor for a row of short grounded elements is then expressed in decibels and evaluated by appropriate series. The results are plotted as gain in decibels against antennæ lengths, wave lengths and spacing of elements. Other calculated relations are given graphically. Experimental results are given for comparison.

Various practical considerations in the operation of arrays are discussed.

A Photo-Electric Integraph. (T. S. Gray, J. Franklin Inst., Vol. 212, No. 1, July, 1931, pp. 102.) (13.5/22106 U.S.A.)

A high narrow beam of light from a vertical line source passes through apertures in two screens bounded by curves, of which the product corresponding is to be integrated and falls on a photo-cell. The intensity of the incident beam is proportional to the product of the corresponding ordinates of the functions. On passing the screens horizontally across the ray the total output of the cell is proportional to the integral between the limits of the horizontal displacement. The error is stated to be within from 2 per cent. to 5 per cent. Examples are given.

Rectifying Effects of Copper Oxide Films. (W. Schottky, R. Stormer and F. Waibel, Z.H.F.T., Vol. 37, No. 5, May, 1931, pp. 175-187. Publication of the Siemens firm.) (13.7/22107 Germany.)

A recent development of photo-electric cells depends on the rectifying action of oxide films. It appears that the effect is influenced by the surface condition of the metal electrode in contact with the oxide film. The effect is considerably increased by etching the metal electrode with nitric acid.

Photo-Cell Theory and Practice. (V. K. Zworykin, J. Franklin Inst., Vol. 212, No. 1, July, 1931, pp. 1-42.) (13.7/22108 U.S.A.)

A clear account is given of the discovery and development of the physical principles. Methods of preparing photo-cells are described and illustrated. The sensitivity at different parts of the spectrum is given graphically for a variety of materials. The distinctive principle of the selenium cell and its analogues is discussed briefly. Methods of amplification are described.

Applications to transmission of sound and visible objects are given in some detail.

Temperature Rating of Engine-Driven Aircraft Radio Generators. (C. B. Mirick and H. Wilkie, Proc. Inst. Rad. Eng., Vol. 19, No. 7, July, 1931, pp. 1175-1183.) (21.03/22109 U.S.A.)

Authors' summary:—

Previously described methods of temperature measurement and computation are applied to engine-driven radio generators in flight. Observed and computed heating curves are shown from which an emission constant has been derived.

Photography

Aerial Cameras and Photography. (Inst. World, Vol. 4, No. 39, July 1931, p. 59.) (14.25/22110 Great Britain.)

Reference is made to an article in the Engineering Journal of Canada, May, 1931, p. 297, which gives a description of the equipment and technique employed in survey work in Canada.

A High-Speed Cine-Camera Giving 2,000-3,000 *Exposures Per Sec.* (E. Huguenard, C.R., Vol. 192, No. 22, January 6th, 1931, pp. 1370-1372.) (14.28/22111 France.)

A film of normal width is used which passes the optical system at the relatively low speed of 3 m.sec. The optical system consists of four small lenses placed side by side covering the width of the film. These lenses are uncovered in turn by a rotating shutter, and 12 photographs 6 × 5 mm. are obtained on the space normally covered by a single exposure.

Explosion Waves and Shock Waves.—Part I., Photography of Bullets in Flight. (W. Payman and D. W. Woodhead, Proc. Roy. Soc., Vol. 132, No. A.819, July 2nd, 1931, pp. 200-213.) (14.28/22112 Great Britain.)

Three special cameras are described and are designated:—

(1) Wave-speed camera,
(2) High-speed wave-speed camera,
(3) Direct photography high-speed camera.

In the first the photographic film is mounted on the outer rim of a revolving drum; in the second on the inner rim, where it is kept in position by centrifugal force; in the third the film is stationary, and the photographic image is moved at high speed over the film by a revolving mirror.

Photographs obtained by different methods are reproduced of tracer bullets and of ordinary bullets in flight through air and water.

Acoustics

Most Favourable Reverberation Time in Halls. (G. v. Békésy, Ann. d. Phys., Vol. 8, No. 7, 1931, pp. 851-873.) (15.20/22113 Germany.)

A vibrating string appears to be damped so that the sound perception falls below the threshold of sensation in a time interval of the order of 0.8 seconds. This time is called presentation time.

The area of the apparent source of sound is limited in space by the restrictions of the ear, and the apparent extent of the source is called the presentation area. The reverberation time is the time in which echo and re-echo sink below the threshold of hearing.

The present paper is concerned with the special problem of the best relations between the acoustical characteristics of a hall and the presentation time and area, with a view to producing the best musical effect of sonority without confusing echoes.

The methods of measurement are of more general interest.

Acoustic Threshold Values. (E. Waetzmann and H. Heisig, Ann. d. Phys., Vol. 9, No. 8, July, 1931, pp. 921-973.) (15.20/22114 Germany.)

The problem is defined and a description is given of recent work, which is often in mutual contradiction on important points. A description is given of a mechanical equivalent ear, which may be standardised in a manner impossible with the human ear.

A hemispherical calibration chamber is described and illustrated by a sketch which gives a practically undistorted acoustical field and agrees completely with the results obtained from a telephone in the position of the ear. Numerous curves of acoustical efficiency are plotted against frequency. The results are stated to agree closely with the values obtained by E. Meyer and to be slightly greater than those obtained by Fletcher and Vogel.

Experiments with Ultra-Sonic Waves. (H. Straubel, Phys. Zeit., Vol. 32, No. 9, pp. 379-383.) (15.20/22115 Germany.)

A quartz oscillator was cut with its edges parallel to the optical and electrical axes and laid upon a polished plate of insulating material. A two-watt valve sender imposed an oscillator frequency of 75,000 hertz producing a wave-length of 4.4 mm. A powder was strewn on the insulator plate and marked the nodes of the air waves.

Ten photographs are reproduced.

Sound-Proofing Cabins. (Aero Digest, Vol. 18, No. 5, May, 1931, pp. 74 and 76.) (15.38/22116 U.S.A.)

A useful descriptive summary, with considerable technical detail, is given of the methods of measuring sound intensity and of applying various materials as dampers to the walls of cabins. The descriptive matter covers tests on absorbent materials, types of wall padding, and tests in two types of cabin, the reduction in sound level being 32 and 28 decibels respectively.

Fire Precautions

Fire Precautions. (F. Kuhn, Z.F.M., Vol. 22, No. 7, April 14th, 1931, pp. 197-206. 216th D.V.L. Report, to be ctd.) (16.05/22117 Germany.)

Precautions against and methods of extinguishing fire are discussed under 25 headings. Tables are given of ignition temperatures for various fuels and of exhaust temperatures for a B.M.W. engine on the test bench. The construction of tanks is discussed and the damage done by shocks is illustrated by photographs. A number of diagrams are given of couplings, and methods of rendering them free from leakage and fracture are discussed. The possibilities of ignition by sparks from the electrical equipment are considered.

Modern Methods of Fire Fighting. (C. B. White, J. Franklin Inst., Vol. 211, No. 6, June, 1931, pp. 757-776.) (16.05/22118 U.S.A.)

Carbon tetrachloride extinguishers are useful only for small incipient fires. Generators producing carbon dioxide gas have a wider application. These devices are rendered much more effective by the production of foam, which keeps the inert gas in the neighbourhood of flame to be extinguished. Types of fire are classified and appropriate extinguishers are recommended.

Gliding

Construction of Sailplanes and Gliders. (A. Lippisch, J. Roy. Aer. Soc., Vol. 35, No. 247, July, 1931, pp. 532-578.) (17.40/22119 Great Britain.)

The expressed aim of the lecturer was to include everything of importance in the last ten years' development. This is carried out by the presentation of scale sketches and photographs of the principal types of glider in flight, accompanied for the most part by dimensioned sketches with empty weight and glider surface. Elementary formulæ are developed showing effective aspect ratio and gliding properties, and the relations obtained are exhibited graphically. A number of details of construction are given, in sketches and photographs.

The extreme logical development of these ideas is expressed in the author's glider "The Ultimate." A discussion followed, and answers were given to 14 participators.

Gliders and Gliding. (W. R. Andrews, Flight, Vol. 23, No. 26, June 26th, 1931, pp. 612-614.) (17.40/22120 Great Britain.)

An elementary discussion is given of the best aspect ratio to select for a glider in order to ensure the best gliding angle. The elementary theory of the transformation of wing profile characteristics for any aspect ratio is given. A numerical example is worked out.

Gliding Competition in Russia. (Stoklitzky, Z.F.M., Vol. 22, No. 11, June 15th, 1931, pp. 335-336.) (17.4/22121 Germany.)

Dimensioned sketches and principal data are given of the competing gliders. Three photographs of machines are reproduced, and one of a glider in flight, showing the nature of the surrounding country.

Meteorological Aspects of Gliding. (F. Entwistle, J. R. Aer. Soc., Vol. 35, No. 246, June, 1931, pp. 423-459.) (17.40/22122 Great Britain.)

A systematic exposition is given of the meteorological conditions which produce rising currents of sufficient vertical velocity to sustain a glider or carry it upward. Special attention is given to the line squall, and a number of observed cases is described in some detail. It is well known that the size of hailstones depends upon the velocity of the upward current in which they are formed. A photograph exhibits the record hailstones which fell in Texas on May 8th, 1926, the diameter of which, $2\frac{1}{2}$ inches, corresponds to a vertical current of about 120 feet per second. Rules for applying the results are laid down and suggestions for further investigation are made.

Carriers

Carrier and Cruiser Design. (Lt.-Com. F. Sherman, U.S. Navy, U.S. Nav. Inst. Proc., Vol. 56, No. 333, November, 1930, pp. 997-1002.) (18.01/22123 U.S.A.)

The duties of a carrier are discussed in relation to attack of and defence against other types of warship. The difficulties in designing a carrier of sufficient speed and defensive power combined with aeroplane carrying capacity leads to a recommendation for the provision of four types of four classes of operation:—

(1) For independent offensive and defensive—roughly, battle cruiser type;
(2) For operations with battle fleet—large carrying capacity, slight defensive qualities;
(3) Scouting carriers—very high speed:
(4) Carrier-cruisers—carrying light bombers and armed with 6 in. guns.

Meteorology

Weather Reporting for the Air Corps. (D. Blake, Air Services, Vol. 16, No. 7, July, 1931, pp. 45-46.) (19.10/22124 U.S.A.)

A descriptive account is given of the operational methods of obtaining meteorological data in the air and of circulating them.

Goodrich De-Icers. (Aero Digest, Vol. 18, No. 5, May, 1931, p. 66.) (19.15/22125 U.S.A.)

The method of removing ice from the nose of a wing by the inflation of tubes in the leading edge has been applied by the Goodrich Co. Diagrammatic cross section sketches illustrate the action of the device.

Picking Up Loads

Device for Picking Up Loads by an Aeroplane in Flight. (Autom. Zeit., Vol. 34, No. 14, May 30th, 1931, pp. 337-338.) (20.10/22126 Germany.)

The load is catapulted in the direction of flight and moves forward with a speed comparable to that of the aircraft when the tackle engages. In this way loads up to 100 lbs. can be taken by the plane in flight without any shock. The device is stated to be in constant use for postal work in America.

Visibility

Navigation Lights. (F. Born, Z.F.M., Vol. 22, No. 9, May 15th, 1931, pp. 253-255.) (21.10/22127 Germany.)

Schemes of arranging navigation lights in aircraft with arcs of visibility and of landing lights with distribution of intensity are discussed with reference to their application in air transport routes.

A Visibility Meter. (M. G. Bennett, J. Sci. Inst., Vol. 8, No. 4, April, 1931, pp. 122-126.) (21.22/22128 Great Britain.)

Author's abstract:—

Very slightly ground glasses are introduced one by one into the field of vision. The number of these glasses which is just sufficient to obscure the object observed is the measure of its visibility. Various precautions are adopted to eliminate sources of error found in other instruments. Applications to meteorology and to illuminating engineering are given.

Fog Penetration. (S. H. Anderson, Air Services, Vol. 16, No. 7, July, 1931, pp. 20-25.) (21.22/22129 U.S.A.)

The elements of the theory of the scattering of light are discussed. The great difference in size between particles in a haze and in a fog and the consequent failure of red light to penetrate the latter is referred to. The only possible development lies in the use of ultra red rays with a wave-length large in comparison with the diameter of the particles. Experiments in this direction are referred to.

Hydro- and Aerodynamics

Modes of Contraction of a Filament at the Entry of a Nozzle. (C. Camichel and P. Dupin, C.R., Vol. 193, No. 2, July 15th, 1931, pp. 102-103.) (22.1/22130 France.)

Four photographs are given, two at low Reynolds numbers showing laminar flow with formation of standing eddies, and two at high Reynolds numbers showing formation of a well-defined boundary layer surrounding the vena contracta. It is found that the contraction is a function of the Reynolds number up to the formation of an apparent surface of discontinuity, after which the contraction remains constant.

The Path of a Vortex in the Neighbourhood of Sources, Sinks, Double Sources and Fixed Vortices, in the Half Plane. (A. Kneschke, Ann. d. Phys., Vol. 9, No. 8, July, 1931, pp. 905-915.) (22.1/22131 Germany.)

The author applies special forms of "Routh's function," obtained by Routh in a paper "Some applications of Conjugate Functions," Proc. Lond. Math. Soc., pp. 73-78, 1818-81, to obtain the equation of the paths of free vortices. These are exhibited graphically.

Vortex Motion Round a Cylinder. (A. Kneschke and S. Matthes, pp. 916-920.) (22.1/22131 Germany.)

The Routh function is applied to find the motion near a semi-circular boss in the x axis, and about a plane projection at right angles to the x axis, of two symmetrically-placed vortices.

Break-up of a Fluid Jet into Drops. (A. Haenlein, Forsch. Geb. Ing., Vol. 2, No. 4, April, 1931, pp. 139-149.) (22.1/22132 Germany.)

The phenomenon, which is of importance in carburation, was examined experimentally. Twenty-four photographs are reproduced showing various stages of the break-up for different fluids. The qualitative explanation of the forces

involved is illustrated by diagrams. A reference is made to Rayleigh's mathematical treatment, taking into account surface tension but not viscosity. For the fluids observed the critical speed is plotted against the jet velocity. In several cases there is sharp discontinuity in the curve. Some elementary dimensional considerations are given.

Break-up of a Fluid Jet into Drops. (C. Weber, Z.A.M.M., Vol. 11, No. 2, April, 1931, pp. 136-154.) (22.10/22133 Germany.)

The experimental stresses discussed in preceding abstract are the subject of analysis in the present paper. Differential equations are formed and solutions are obtained in Bessel functions. The rational nature of the assumptions made, and the agreement with experiment, indicate that a truly physical theory of the phenomenon has been established.

The Viscous Layer Associated with a Circular Cylinder. (J. J. Green, Phil. Mag., No. 75, July, 1931, pp. 1-41.) (22.1/22134 Great Britain.)

The equations of viscous fluid motion are reduced to approximate form by Prandtl's device of assuming the existence of a narrow boundary layer in which transitions take place from zero motion at the surface of a cylinder to potential motion outside it. The equations are reduced to non-dimensional form in the usual way, and a solution in series is assumed. Using observed experimental values, successive numerical approximations for the coefficients are constructed. As with other solutions propounded by German writers, the validity of the approximations certainly fails at the branching point where the initial stream-line leaves the surface of the cylinder. The predicted branching point is claimed to be in satisfactory agreement with experiment.

The computations are laborious, and it is difficult to assess the relative convenience and accuracy of this method as compared with other approximate solutions.

Superposed Turbulence. (E. G. Richardson and E. Tyler, Phys. Zeit., Vol. 32, No. 13, July 1st, 1931, pp. 509-517.) (22.1/22135 Germany.)

Reference is made to the work of Prandtl, v. Karman, Schiller, and others, in obtaining approximate forms of equations of viscous fluid motion appropriate to the disturbed motion in a pipe or past a barrier or periodically disturbed. Some elementary results are given, subject to boundary conditions which are not reproducible in the laboratory. In their experimental work the authors have superposed periodic axial motion by the oscillation of a concentric sliding mouthpiece.

With regard to the problem of flow past an obstacle, the turbulence was set up by the vibration of a strip close to the nose of a stream-lined strut, or by the oscillation of a cylinder.

The velocity fields are plotted as explored by hot wire instrument, and six samples are given in three figures, the upper half of the figures showing the natural flow and the lower half the effects of artificially superposed turbulence.

Stability and Turbulence of Fluid Motion. (W. Heisenberg, Ann. Phys., Vol. 74, No. 15, 1924, pp. 577-627.) (22.1/22136 Germany.)

The author summarises the position of the problem (1924), and, following Rayleigh, forms non-dimensional differential equations. Methods of integration are developed. Boundary conditions are examined. Stability under external disturbance is investigated and a condition is obtained for critical stability. The result is obtained that Rayleigh's substitution of a polygon for a curvilinear disturbance of velocities is not permissible. Further, if a distribution of velocity is unstable in a perfect fluid, it is likewise unstable in the presence of viscosity,

beyond a certain Reynolds number. A critical wave-length exists for a disturbance which is damped out.

A discussion of turbulence follows. It is not possible to summarise the argument without a detailed mathematical discussion.

Circulation Round a Rotating Cylinder in a Viscous Fluid. (E. G. Richardson, Phil. Mag., Vol. 11, No. 74, June, 1931, pp. 1215-1220.) (22.1/22137 Great Britain.)

The velocity field in the neighbourhood of a rotating circular cylinder was explored with a hot wire instrument. It was found that the velocity fell off more rapidly near the cylinder than for the irrotational flow which would be the final steady distribution for a long cylinder rotating in a large mass of viscous fluid. The suggestion is made that instability near the cylinder causes turbulence and gives a higher effective transference of vorticity from the surface of the cylinder outwards.* Experimental results are exhibited graphically and an empirical expression is obtained.

Flow of Water Through Circular Concentric and Rectangular Tubes. (F. C. Lea and A. G. Tadros, Phil. Mag., Vol. 11, No. 74, June, 1931, pp. 1235-1247.) (22.2/22138 Great Britain.)

The elementary formulæ for steady laminar flow are worked out for circular and concentric circular tubes. Experimental values are plotted logarithmically and exhibit the breakdown at the critical Reynolds number and the subsequent flow with the resistance proportional to the velocity raised to the power of 1.73. A small core appears to have the effect of setting up and maintaining turbulence at much lower Reynolds numbers than for circular tubes without core. The assumption of slip at the core surface is introduced. Formulæ are given, without demonstration, for mean velocity with laminar flow in the rectangular tubes.

Influence of Viscosity on Measurement of Velocity with Venturi Tubes. (H. Peters, Z.F.M., Vol. 22, No. 11, June 15th, 1931, pp. 321-323.) (22.4/22139 Germany.)

The ideal relation between pressure drop, density and velocity contains a multiplying factor K, which would be constant for an ideal fluid, but varies considerably for a viscous fluid.

Values of the factor are determined under a wide range of conditions of temperature and pressure for the Bruhn double venturi tube and the results are shown graphically. An elementary theory is laid down in which the effects of compressibility are considered. The readings of an ordinary pitot tube are increased 6 to 13 times.

Secondary Eddies Behind an Obstacle in a Stream. (E. Crausse and J. Baubiac, C.R., Vol. 192, No. 24, June 15th, 1931, pp. 1529-1531.) (22.4/22140 France.)

Circular cylinders of diameters between 8mm. and 100mm., and of axial length 3mm., were placed in a stream or moved in a mass of fluid. A photograph shows the initial vortex detached and another shows secondary eddies forming along the boundary of the turbulent wake. The period of the secondary eddies is plotted against the velocity for a cylinder of 15mm. diameter.

Recording of Period of Eddies by Hot-Wire Instrument. (E. Crausse and J. Baubiac, C.R., Vol. 192, No. 22, June 1st, 1931, pp. 1355-1357.) (22.4/22141 France.)

The periods of alternating eddies behind an obstacle recorded by a hot wire instrument are plotted for water and for a sugar solution with a coefficient of

* This suggestion, illustrated by sketches, was put forward by Foppl, in discussion, at the International Congress for Applied Mechanics at Zürich, 1926.

viscosity four times as great. On transforming for dynamic similitude the experimental points for both fluids are closely grouped along a unicursal curve. The record of the periodicity is reproduced and shows great uniformity at about 0.8secs. A record of the periodic effect of secondary eddies is also reproduced. The periodicity is less regular than in the previous example, but is reasonably uniform at about 0.017secs.

Experiments in Sucking Away the Boundary Layer of a Wing. (O. Schrenk, Z.F.M., Vol. 22, No. 9, May 15th, 1931, pp. 259-264.) (22.6/22142 Germany.)

Plates with punched orifices were fitted to the surface. The distribution of the orifices and the position of the plates is shown diagrammatically. The test results for lift, drag and moment are given graphically, and are compared with the calculated results of hydrodynamical theory. Comparison with previous results shows a marked advance in increasing the lift coefficient by this method. The observed incidence in the latest experiments also approach much more nearly the theoretical incidence.

Resistance of the Air to Ballistic Speeds. (Gabeault, C.R., Vol. 192, No. 25, June 22nd, 1931, pp. 1630-1633.) (22.4/22143 France.)

The formation of a series of ring vortices behind an ogival shell is assumed. Using an elementary analysis analogous to v. Karman's two-dimensional theorem, a formula for the resistance is obtained showing a discontinuity at twice the speed of sound. Comparison with experimental results obtained by others shows points of quantitative agreement.

(The fact that such a regular series of ring vortices is not necessarily formed, or if momentarily formed is not stable, may not invalidate the result as some sort of time average).

Lift on a Flat Plate Between Parallel Walls. (L. Rosenhead, Proc. Roy. Soc., Vol. 132, No. A.819, July 2nd, 1931, pp. 127-152.) (22.4/22144 Great Britain.)

Methods of conformal transformation are applied to find the lift upon a flat plate between walls. The results are obtained in Weierstrass and Theta functions. An error is pointed out in a paper by Sasaki in applying a method of Villat's. Numerical values are tabulated to various approximations and compared with Glauert's values, which are shown to be useful first approximations.

Compressibility Effects in High-Speed Air Flow. (S. G. Hooker, J. Roy. Aer. Soc., Vol. 35, No. 247, July, 1931, pp. 645-647.) (22.4/22145 Great Britain.)

A summary is given of the restricted progress made in taking into account the compressibility of the air in aerodynamical relations.

Wind Pressure on a Model of a Mill Building. (H. L. Dryden and G. C. Hill, B. St. J. Res., Vol. 6, No. 4, April, 1931, pp. 735-763.) (22.4/22146 U.S.A.)

The model building, 48 ins. long, 24 ins. wide, and $12\frac{3}{4}$ ins. high, was placed in the 10 ft. tunnel and pressure measurements were made at 388 orifices. Charts of pressure distribution under different wind directions are reproduced. Local pressures were as high as twice the mean pressure.

Miscellaneous Unclassified

The Art of Metallography. (F. F. Lucas, Bell Tele. Lab., B.565, June, 1931.) (22147 U.S.A.)

The optical principles of metallography are discussed and in particular the application of ultra-violet rays to obtain finer detail is considered. Twenty micro-photographs are reproduced.

Application of Statistical Method in Engineering. (W. A. Shewhart, Bell Tele. Lab., B.560, May, 1931.) (22148 U.S.A.)

The paper indicates briefly the sort of problems an engineer can solve by the statistical method which he could not solve without it. The examples selected show test results as distributed on a frequency curve instead of as an undistributed set of variations from the mean value.

Measurement of Dust in Smoke Gases. (E. Zimmerman, Z.V.D.I., Vol. 75, No. 16, April 18th, 1931, pp. 481-486.) (22149 Germany.)

The Berlin Electricity Works have standardised the measurement of the dust in flue gases by filtering a portion of the gases through a glass wool filter and measuring the increase in weight. Certain precautions are necessary in designing the pipe connection between the filter and the main gas passages, and in obtaining representative samples and correlating with the total delivery of flue gas.

Visibility from Aircraft. (D. Huntingdon, Aero Digest, Vol. 18, No. 6, June, 1931, p. 52.) (22150 U.S.A.)

The importance of visibility to commercial craft is discussed in a non-technical manner. The article is illustrated by the totally inadequate representation of the arc of view in the plane of symmetry only.

Field of View. (G. Kurtz, Z.F.M., Vol. 22, No. 6, March 28th, 1931, pp. 167-176, 215th D.V.L. Report.) (22151 Germany.)

The comparative fields of view of the seven aircraft reproduced show remarkable differences in the total area obscured. The importance of measuring the field of all types of aeroplanes for comparative purposes is emphasised. The methods are substantially those employed in this country in 1917, but since discontinued.

Production and Measurement of High Vacua. (S. Dushman, J. Franklin Inst., Vol. 211, No. 6, June, 1931, pp. 689-750.) (22152 U.S.A.)

Recent advances in production and measurement of high vacua are summarised, and some recent results are tabulated. The lowest value obtained is about 10^{-7} mm. of mercury, roughly equivalent to 10^{-6} dynes per cm.^{2} A large number of applications are discussed.

Acetylene Storage. (The Gas Accumulator Co., British Patent No. 339899, May 30th, 1929.) (22153 Great Britain.)

The cylinders for holding the acetylene are filled with a cemented block of pumice, artificial slag, or other material having the same coefficient of expansion by gas absorption and heat as cement.

Night Air Mails. (Carl Florman, J. R. Aer. Soc., Vol. 35, No. 246, June, 1931, pp. 460-488.) (22154 Great Britain.)

The author is the Managing Director of the A.B. Aerotransport, Stockholm, and his analysis of conditions of operation of night air mails in Europe and America indicates the importance of various factors as they appear to an experienced administrator of flying services.

REVIEWS

Principles of Flight

E. A. Stalker. Ronald Press. $6.00.

This is a work which can be thoroughly recommended. It deals adequately with the modern theories of aerodynamics associated with the name of Lanchester and Prandtl and deals with stability and controllability, and though these matters are discussed from a theoretical standpoint there are many curves given from wind tunnel tests which help to keep the student's attention on the practical side of the theoretical discussions. The more practical subject of aeroplane performance is hardly so adequately treated, certain modern developments such as supercharged engines being left out of account.

One cannot help regretting on reading an American book such as this that there is not better co-ordination between the two countries in the matter of the symbols used, and to those used to English symbols our attention is often distracted by having to remember that letters frequently do not carry the same meaning in America as in England. I am also in some doubt as to some of the standard figures given in Table 21. Standard temperature absolute is not 518.5°F. on this side of the Atlantic but 459.2°F. Also the standard gas constant for air is not quite correct according to English ideas, being given as 53.33089 instead of 52.20.

However this may be, I have not previously perused a work in which the modern aerodynamic theories are more clearly explained and it is well worth reading by those interested in the subject.

Structures

J. D. Haddon. Gale and Polden. 6/-.

Mr. Haddon's book on Structures is excellent of its kind and should fulfil adequately the purpose for which it was written; the matters dealt with are well explained and the language is clear and explicit. The scope of the work is good and is sufficiently up-to-date to include a discussion on steel structures, in fact, the only addition that might have been usefully made is a discussion on airscrews from a structural standpoint. I do not however quite follow the reference to " Watten " girders in connection with rib design, but this is possibly an error for " Warren." But Warren is hardly *N* bracing.

Personally, I think that it is a pity in a work of this kind that calculus proofs should be so completely omitted. Admitting that many of those for whom the book was written have no acquaintance with the subject, yet it would seem probable that earnest students would be stimulated by the inclusion of such proofs to a determination to know how they were derived. This would certainly be to their advantage and would also save Mr. Haddon from having to describe the clumsy method of obtaining the moment of inertia of a fly-wheel, as given on page 8. I am afraid that the author may be encouraging the mistaken but common notion that calculus is a subject which can only be understood by those who are possessors of abnormal brain power.

However this may be, this work can be thoroughly recommended and every student who gives it adequate attention should be well grounded in the subjects dealt with.

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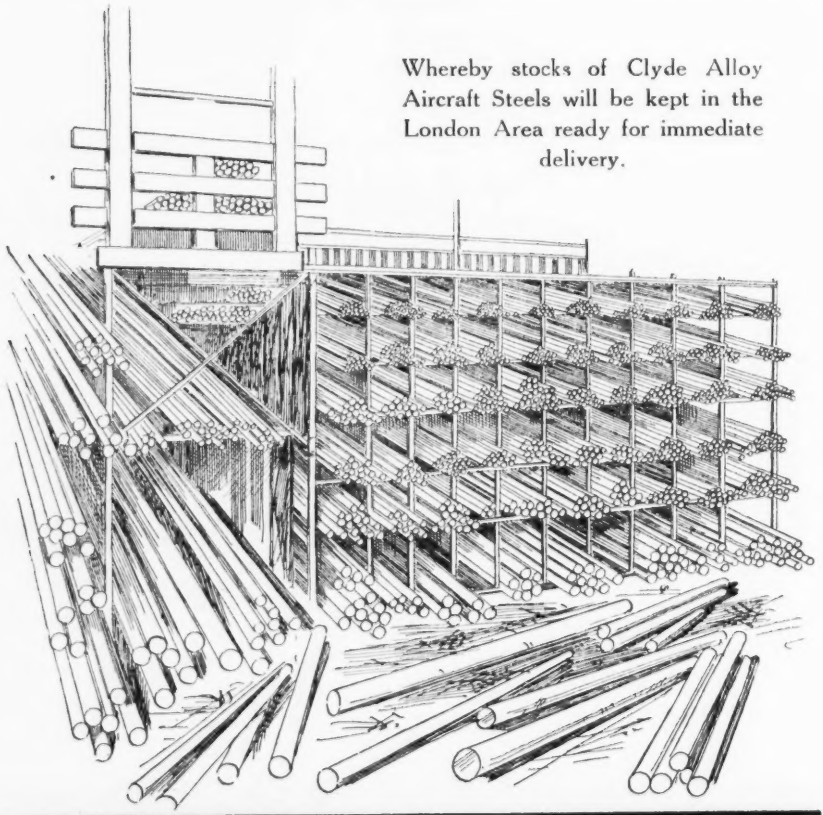

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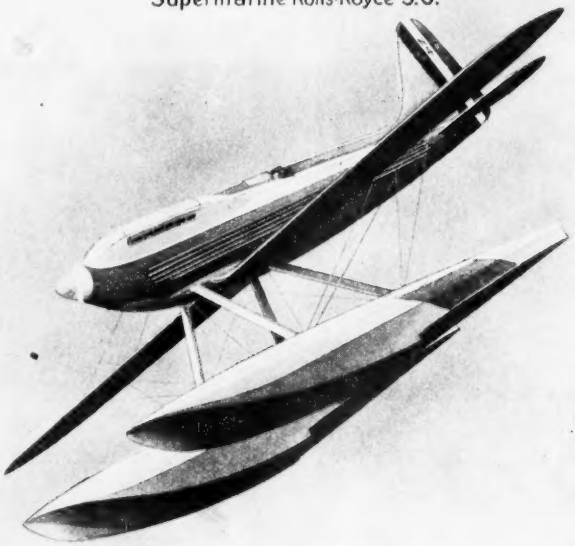

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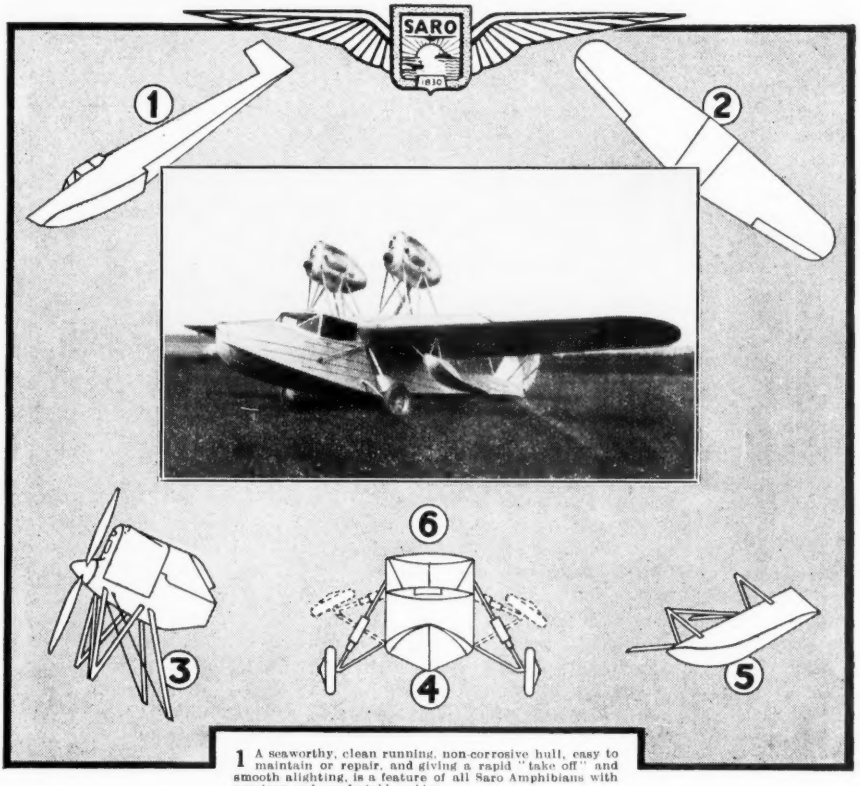

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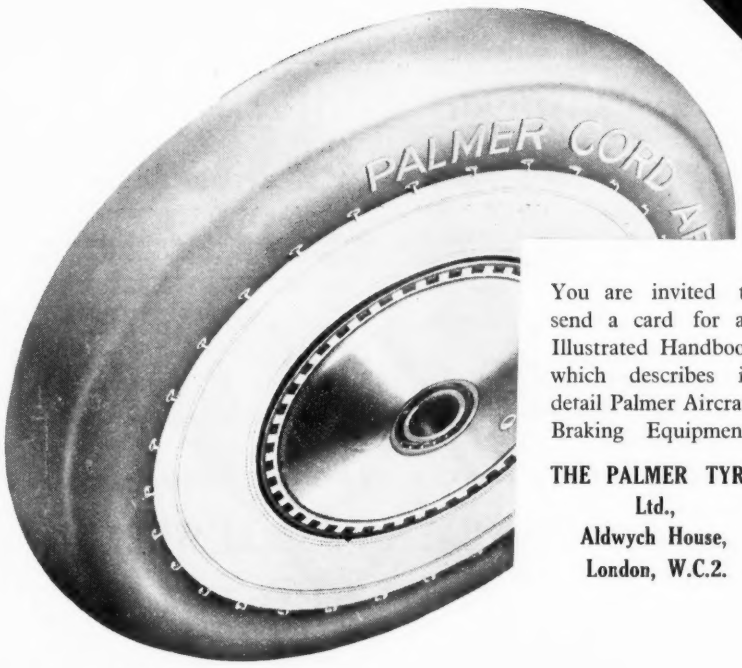

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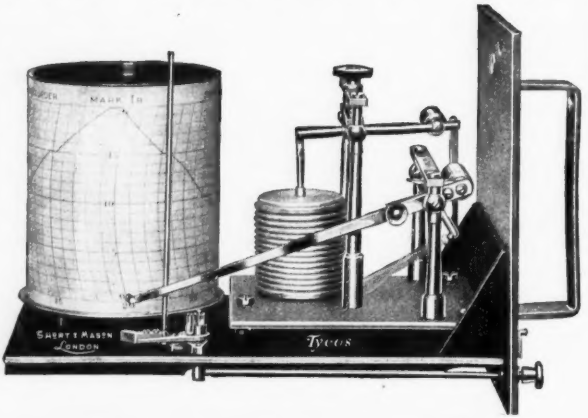

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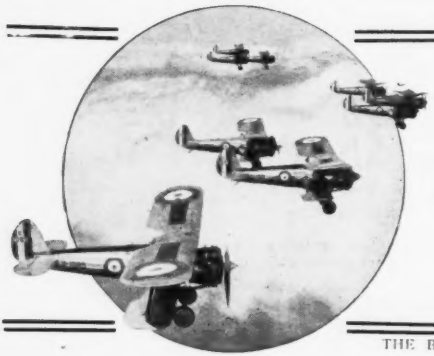

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